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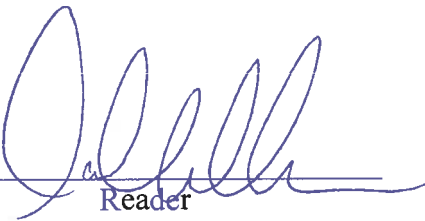
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Confidence Intervals in a High School

Statistics Class

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Abstract

In order to be a functioning member of society, it is important that a citizen understands basic statistics. One key topic in particular is confidence intervals. There are confidence intervals in everyday life, no matter if you are a person in college or just a person in society. Many claims about population rely on a sample and the statistics of confidence intervals. For example: We believe, with 95% confidence, that $40\% \pm 3\%$ of Americans will approve a new law by Congress, meaning that we are 95% confident that any sample formed in this way will have a proportion or mean that falls in the confidence interval. For my thesis project, I created a lesson that can be used to help teach high school students to understand what a confidence interval is and how to use it. This lesson uses multiple strategies to help students understand how to create and interpret a confidence interval. The lesson also follows the Common Core State Standards for statistics that relate to confidence intervals. The hope is that by informing high school students about how to interpret and use a confidence interval, they will be able to understand the statistics they will encounter on a regular basis.

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1 What is a Confidence Interval?

When you are reading a research document or when you read a poll, you will most likely hear something to the effect of: “The research found, with a confidence of 95%, that the percentage of people that would choose Kik-Kat over Reese’s is $40\% \pm 3\%$.”

What are the researchers actually trying to say?

1.1 Sampling

Before we delve into confidence intervals, we need to build a basis and start with a sample. Unless the population is small, it is nearly impossible for a researcher to interview or test every single person in the given population. A population is “the complete set of observations a researcher is interested in” [8]. Instead, a researcher needs to create a sample, “a subset of a population” [8], in order to make a statistical judgment about the population.

When deciding on a sample it is important that the elements of the sample are chosen at random. This will allow for less bias in the sample. When there is bias in the sample, the results of the statistical analysis will be a less accurate representation of the population in question.

1.2 Sampling Distributions

Once a sample has been made, the next step is to make the sampling distribution. A sampling distribution is the “distribution of values taken by a statistic in all possible samples of the same size from the same population” [16]. It is often impossible for a statistician or anyone looking at a population to know the true mean of the population, μ . Therefore, we will take samples of size n . When looking at the sampling distribution and the mean of the sampling distribution of $\mu_{\bar{x}}$, we can say that $\mu_{\bar{x}} = \mu$, where \bar{x} is the mean of the simple random sample of size n of the population and

$\mu_{\bar{x}}$ is the mean of many samples taken of the same size. We can also state that the standard deviation of the sampling distribution is $\sigma = \frac{s}{\sqrt{n}}$. The standard deviation of the sample, s , describes the spread of the sample. Example 1 and 2 represent different sampling distributions.

- **Example 1:** A researcher is tasked with finding the mean height of all men in the state of Montana. He does this by taking random samples of 30 means. Every time he takes a sample of the 30 height he finds the mean of the 30 heights and plots it. He does this over and over until he has done 150 samples. The plot that he has created is a sampling distribution. He found looking at the sampling distribution that $\mu_{\bar{x}} = 5.9$ feet.
- **Example 2:** A cereal company would like to know how many raisins are in a box of their cereal. To do this, the company hires a research team to take random samples of size 40 boxes from the population of all the cereal made in the factory. The research team found the mean of each sample of size 40 and plots that mean on a diagram. When the researcher finishes he looks at his plot and finds that $\mu_{\bar{x}} = 50$ raisins.

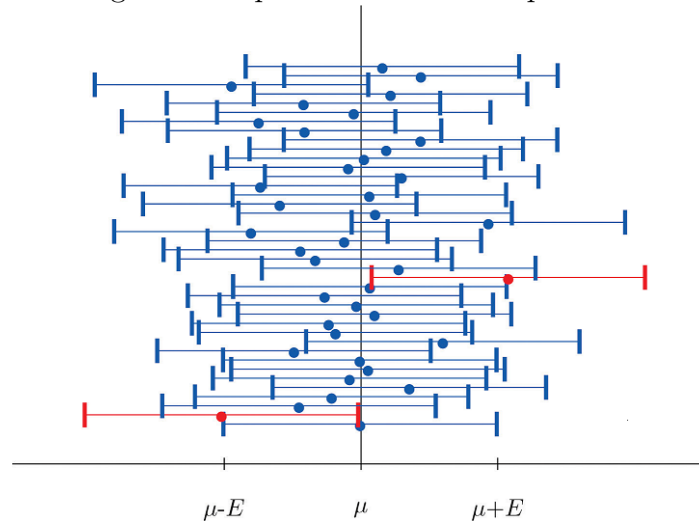
One of the most important theorems in statistics that allows us to make inferences about a population from a sample is the Central Limit Theorem. The Central Limit Theorem states that “[i]f random samples of n measurements are repeatedly drawn from a population with finite mean μ and standard deviation σ , then, when n is large, the sample means will have a distribution which is approximately normal (bell-shaped), with mean $\mu_{[\bar{x}]}$ and standard deviation of $[\sigma] = \frac{s}{\sqrt{n}}$ ” [9].

1.3 Creating a Confidence Interval

Now that we have a sampling distribution, we are able to start analyzing the distribution. When creating a sample, there is error built in due to the nature of sampling.

This means that we cannot assume that the mean of the sample is the same as the mean of the population. Figure 1 shows how when we look at samples created in the same way, there are going to be variations and thus we need to create a confidence interval about our samples. As we make more and more samples of similar sizes, we will have a confidence as to how many of them will contain the actual mean of the population.

Figure 1: Representation of Sample Error



To begin making a confidence interval we need to decide which confidence level we want. Three common confidence levels are 90%, 95%, and 99%. The main confidence level that is used in the project is 95%. In order to create a confidence interval we need to find the critical z-score, or the z_{crit} , for the confidence level for which we are looking. The z_{crit} will tell us how many standard deviations we are away from our mean. In order to do this, we look at a z-table to find the z-test statistic that corresponds with each probability. Table 1 shows these z-critical values.

Table 1: Z-critical values for Confidence Intervals

Confidence Level	z_{crit}
90%	1.645
95%	1.96
99%	2.575

When we create a sample, we will have a standard deviation of our sample, $s_{\bar{x}}$. When creating a confidence interval we will use this to find the standard error of the sample mean. This is calculated by taking the standard deviation of the sample and dividing it by the square root of the sample size, equation 1.

$$SE = \frac{s}{\sqrt{n}} [16] \quad (1)$$

Using the z-critical value and the standard error, we are able to find the margin of error for our sample. Equation 2 shows how we find the margin of error.

$$ME = z_{crit} * \frac{s}{\sqrt{n}} [16] \quad (2)$$

Now that we have created a margin of error, we are finally able to create the confidence interval. Equations 3 and 4 show the set up of the confidence interval.

$$CI = \bar{x} \pm ME \quad (3)$$

$$CI = \bar{x} \pm z_{crit} \frac{s}{\sqrt{n}} \quad (4)$$

One important concept that this method assumes is that our $n \geq 30$. This allows us to use a z test statistic because when the sample size is greater than 30 it is assumed that the sample distribution will be normal. When the sample size is less than 30 we cannot assume that it is normal. Therefore, we have to use the student t-distribution. The only thing that this changes is that we have to use a t_{crit} instead of a z_{crit} to calculate the confidence interval. The t_{crit} value will also tell us how many standard deviations we are away from our mean, but this time on the student t-distribution.

1.4 Interpretation of the Confidence Interval

Now that we have created the confidence interval, it is important that we understand what it is that we are saying. To interpret a $C\%$ confidence interval for an unknown parameter, say, “We are $C\%$ confident that the interval from _____ to _____ captures the [parameter in context]” [16].

Example: In a study, a researcher found that that the sample mean, \bar{x} , of scores in bowling to be 159. This sample was taken of 120 adults ages 30-40. He also found the standard deviation to be 5 points. Find and interpret a 95% confidence interval.

- Now imagine a study done by the University of Montana that found the mean to be 163. Compare your confidence interval with these results.

This data is a classic example as to what a confidence interval is used for in daily life. In order to create the confidence interval, we must first find the margin of error, ME . Recall from equation 2 what is needed in order to find the margin of error. We are looking for a 95% confidence interval and we have a sample size of over 30; therefore, we can assume that the sampling distribution is normally distributed. Thus, we can use the z_{crit} value of 1.96. We also need to find the standard error of the sample distribution, using equation 1.

$$SE = \frac{s}{\sqrt{n}} = \frac{5}{\sqrt{120}} \approx .456$$

We can now use both the standard error and the z_{crit} to find the margin of error for this study.

$$ME = 1.96 * .456 = .8892$$

Now that we have the margin of error we can find the 95% confidence interval:

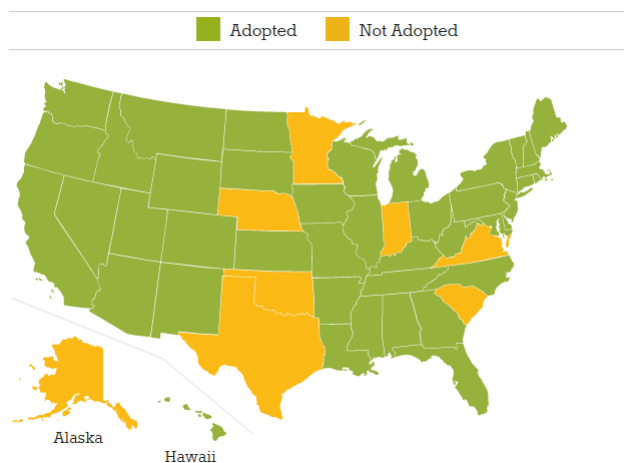
$$CI = 159 \pm .8892$$

With this confidence interval we are able to say that we are 95% confident the actual mean of bowling scores of adults ages 30-40 is between 158.11 and 159.88. This also means that we are also 95% confident that the true mean of the population will fall between these same two numbers. Therefore, when we look at the mean found by the University of Montana study, we can see that their mean is outside of our range. Because we are only 95% confident about our interval, there is a 5% chance that the mean could fall outside of our range. Therefore, we have evidence that does not support the study at the University of Montana. We may be wrong because there is still a chance that the true mean falls outside of our range, however this is only a 5% chance where we have a 95% chance that the mean will fall in our range.

2 Analysis of Confidence Intervals in Common Core

Common Core is a large topic for discussion in the United States education system. “The Common Core is a set of high-quality academic standards in mathematics and English language arts/literacy (ELA)” [1]. “Forty-two states and the District of Columbia...have adopted the Common Core State Standards” [15]. Figure 2 shows the states that have adopted the Common Core State Standards (CCSS).

Figure 2: Map of states that have adopted the Common Core State Standards[15]



As we can see there are seven states that have not accepted common core. Minnesota has only accepted the ELA standards. The Common Core State Standards for Mathematics are separated out by grade until high school. The high school standards are separated out by class. There is a section on the Common Core standards for Probability and Statistics.

In these statistics there are no standards that specifically mention confidence intervals. However, if we look at the standards, we can see that confidence intervals are built in. One of the standards for Statistics and Probability is 'Making Inferences & Justifying Conclusion.'

- CCSS.MATH.CONTENT.HSS.IC.B4
 - “Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling” [6].

Whenever we take a sample and make an estimation of a population mean or proportion and then develop a margin of error, we will have a confidence interval built in. Taking the mean or the proportion and adding and subtracting the margin of error is exactly what a confidence interval is. Therefore, even though the standard does not specifically state that the students need to understand what a confidence interval is, the standard gives us enough information to build a confidence interval and thus the students should be developing an understanding of what a confidence interval means.

- CCSS.MATH.CONTENT.HSS.IC.B6
 - “Evaluate reports based on data” [6].

This standard builds the importance of having students understand confidence intervals. When students are given data in the real world, there will most likely be a

confidence interval built into the report. Unless a student knows how to interpret a confidence interval, they will be unable to make appropriate analyses of the data. If they know how to read and understand a confidence interval, they can see if the data they have been presented is statistically significant or even logically sound. Without this knowledge, the students entering into college or the real world will not be able to make sound judgments on the data presented to them.

As well as having standards that are just on statistics, the Common Core State Standards also have Standards for Mathematical Practice. These are expertise that teachers of all levels should seek to develop in their students. A study in confidence intervals provides an opportunity to build up these mathematical practices, particularly MP 3.

- CCSS.MATH.PRACTICE.MP3
 - “Construct viable arguments and critique the reasoning of others” [13].

The main purpose of confidence intervals is to look at what someone else has done and see if what they concluded makes sense. A student who knows how to construct a confidence interval can construct an argument about what the results show. Also, if a student knows about confidence intervals, he/she would be able to look at someone’s results and critique whether or not what they have concluded is sound statistics or not. They could do this in multiple ways. The first would be to do a study of their own and see if their results hold up to what the other results stated. Another way of doing this, is to look at other studies and see the results of those studies and see how they compare to each other. If a student doesn’t have any background in confidence intervals, they will not be able to make “critiques about the reasoning of others” [13] in the context of inferential statistics.

3 Creating a Mathematical Lesson

For this thesis, I created a lesson plan for teaching Confidence Intervals. I then taught this lesson to the Elementary Statistics class at Carroll College. The reason I chose to teach this to the students in this class is due to the fact that this class is very similar to that of an AP Statistics class that a student in high school would take.

Before the lesson was developed, I read the book 5 Practices for Orchestrating Productive Mathematics Discussion. In this book there are five practices outlined to support a meaningful mathematics discussion in the classroom, and should be done in this order when preparing and executing a lesson.

- Anticipating
- Monitoring
- Selecting
- Sequencing
- Connecting

The first of these is anticipating. This means to think about any possible questions that the students may or may not ask about the lesson. “Anticipating students’ responses involves developing considered expectations about how students might mathematically interpret a problem, the array of strategies - both correct and incorrect - that they might use to tackle it, and how those strategies and interpretations might relate to the mathematical concepts, representations, procedures, and practices that the teacher would like his or her students to learn” [12]. This requires the teacher to do the problems as many ways as possible and determine what errors the students could make. It also requires the teacher to see if there is more than one way to solve the problem. Even if there are multiple ways to solve the problem, it is important

that the teacher show how this answer will be correct but it doesn't show that they know the mathematical concept presented in the lesson. This step also requires the teacher to think about what possible answers the students will give to the questions presented in class. If the teacher can prepare in this way, they will be able to make a more meaningful mathematical discussion.

The next practice in creating a mathematical discussion is monitoring. "Monitoring student responses involves paying close attention to students' mathematical thinking and solution strategies as they work on the task" [12]. The practice requires a teacher to be an active observer in the classroom. A teacher needs to be taking notes as to what each group is doing. Using the processes that were found during the anticipating step, a teacher can make comments as to which groups or students are doing which method. The classroom teacher must also be aware that students may discover other methods and should use this time to look at and record other methods discovered by the students. Using this information, the teacher can then move onto the next practice: selecting

"The selection of particular students and their solutions is guided by the mathematical goal for the lesson and the teacher's assessment of how each contribution will contribute to that goal" [12]. When the teacher knows what each group/student has done in order to solve or answer the mathematical question, he/she can decide which groups he/she would like to present or discuss their information. They could also use this time to look at the students' solutions and review the different possibilities together as a class and see where the student/group went with their work. To help make sure that the student doesn't feel like the teacher is making fun of an answer, the teacher can not tell the class which group/student had which answer.

When the teacher has made the decision of which students/groups to use, he/she can move onto sequencing. "By making purposeful choices about the order in which students' work is shared, teachers can maximize the chances of achieving their math-

emational goals for the discussion” [12]. Using the solutions that the teacher chose in the selecting phase, the teacher can decide what order to go through the different solutions. This way the teacher can work through the students’ answers and have the students compare different solutions. While the students are comparing the different answers, they are able to see errors that may have been made. This will help the students understand the topic better because they are using critical thinking when looking at each solution.

The final practice is to connect the solutions that students have given to the other solutions. “The teacher can help students make judgments about the consequences of different approaches for the range of problems that can be solved, one’s likely accuracy and efficiency in solving them, and the kinds of mathematical pattern that can be most easily discerned” [12]. A teacher can take the ideas that the class came up with and help analyze the different solutions to see why the different errors were made. Using this information, students can hopefully understand the mathematical concepts better. The connection practice also allows the teacher to help the students move to a deeper level of thinking. With a deeper level of thinking, students will enhance their critical thinking skills and be able to make better judgments in the future.

4 Lesson Plan For Confidence Intervals

Now that we have looked at the five practices for meaningful mathematical discussion, we can review the lesson plan that was created for this thesis. This lesson is meant to be used after a lesson on Sampling Distribution. This is also a two day lesson.

DAY 1

The first part of this lesson is to do the anticipating step. This will be different for every teacher and every class. The teacher will need to make these judgments based

upon his/her class. In this lesson the students will engage in problems that require them to make confidence intervals using a normal distribution. They will need to find the critical z-scores and create margins of error from the data. They will finally create confidence intervals based on the information given to them. It is important that the teacher thinks about every question that a student may ask about confidence intervals. Some examples are:

- What does margin of error mean?
- How do we represent sample error?
- From where do the critical z-scores come?
- What does a confidence interval tell us?

Using these questions, the teacher can then prepare for the class. This lesson uses an online technology called Nearpod. Nearpod can be used on a phone via an app or online. The teacher can prepare a slide show that will have questions that are open ended, drawing, or multiple choice. The students can either use the Nearpod app or the Nearpod website to connect to the class session. In this lesson we will recommend that students use made up names. This will help save the students from the embarrassment giving a wrong answer.

The lesson will begin with students logging onto Nearpod. Once everyone is ready the teacher will send a Draw It question:

- Draw and label the normal distribution.

While the students are working, the teacher can monitor what answers are coming in, or walk around the class and see what different students are drawing and discussing. Once all of the students have finished, the teacher can select the drawings that he/she would like to review. By sequencing the drawings, the teacher can get students on the right picture.

Once this is finished, have the students answer the following open ended questions:

- Why does the Normal Distribution look like this?
- What does a standard deviation tell you?

Again, using monitoring, selecting, and sequencing the teacher can prepare and guide the students to arrive at the answer to the questions.

Now it is time to go into the discovery portion of the class. Once the the questions are done the teacher should move onto the discussion of how we created this normal distribution.

- When you take many random samples from a population and find the mean of each of the samples and plot them on a chart, the shape of the graph will be a normal distribution.

Then it is time for another open ended question. Because this lecture has happened after the students have learned what Standard Error is, this question is just a recall question from information that they already have.

- What is Standard Error?

The next step is to review the equation for Standard Error. The teacher needs to make sure that he/she mentions that this Standard Error is the standard deviation of our sampling distribution. It is time to go through an example.

“Studies have shown that the random variable X, the processing time required to do a multiplication on a new 3-D computer, is normally distributed with an unknown mean μ and standard deviation 2 microseconds. A random sample of 16 observations is to be taken (i.e. 16 random multiplications will be performed and the time that it takes to perform each one of them will be annotated). Construct a 95% confidence interval as to where this value of X falls.”[5]

Table 2: 3-D computer multiplication[5]

42.65	41.63	46.50	43.87
45.15	41.54	41.35	43.79
39.32	41.59	44.37	43.28
44.44	45.68	40.27	40.70

The teacher will then step the students through the process of creating a confidence interval. The first step is to find the mean and standard deviation of the data set. The problem statement gave us the standard deviation, 2. Then we need to find the mean of this data set. The mean data is 42.9 microseconds. From this we can begin to form the confidence interval. The problem statement tells us that we want to find a 95% confidence interval. This means that we are going to be 1.96 standard deviations away from our mean. We then need to find the standard error. It is important to remind the students that the standard error equation is:

$$SE = \frac{s_{\bar{x}}}{\sqrt{16}} = \frac{2}{\sqrt{16}} \approx .526$$

Using this and the fact that we know we are 2 standard deviations away from our mean, we can find the margin of error to be:

$$ME = 1.96 * .526 = 1.053$$

Using this information we can finally create the confidence interval:

$$42.9 \pm 1.053 = (41.846, 43.953)$$

When the students have found the confidence interval, the teacher will then send an open ended question:

- What does this confidence interval mean in context?

- This confidence interval means that we are 95% confident that the true mean of the population will fall between (41.846, 43.953).

Then it will be time to send the students to work together in groups or partners.

They will work on the following problem:

- A study at UM showed that the average weights of people in the United States to be 168 lbs with a standard deviation of 30 lbs. Other studies have shown that the average weight of people in the United States is normally distributed with a standard deviation of 30 lbs. Your test showed that a random sample of 160 weights were taken. The mean of these weights was given as 156 lbs. Construct a 99% confidence interval as to where this average weight value is. (Does the schools data match your conclusion?)
- By using the 68-95-99.7 rule, we know that 99% holds 2.58 standard deviations.
- This means that we can find the standard error to be

$$\frac{30}{\sqrt{160}} = 2.372$$

- We can then use this information to find the margin of error.

$$ME = 2.58 * 2.372 = 7.115$$

- $CI = 156 \pm 7.115 = (148.88, 163.11)$
- UM showed that they had a mean of 168 with a standard deviation of 30. Our confidence interval shows that their mean is outside of our range. Thus, we can say that they may be incorrect. Because we created a 99% confidence interval, we can say with high confidence that we have evidence to believe that they did not find the true mean of the population.

DAY 2

Day 2 is to look more into concrete examples of using confidence intervals as well as create a confidence interval with a normal distribution and a student t-distribution. The final part of the Day 2 lesson will look at confidence intervals in daily life. This part of the lesson will require students to have a computer that has Excel or a graphing calculator.

Begin class by presenting students with the problem:

- You sample 36 apples from your farm's harvest of over 200,000 apples. The mean weight of the sample is 112 grams (with a 40 gram sample standard deviation). Construct a 95% confidence interval as to what the actual weight of the 200,000 apples is.[\[7\]](#)

We know that because we have a sample size that is greater than 30, we can use the normal distribution. In this case we know that $\bar{x} = 112$. The standard deviation of this sampling distribution is $s = 40$. With this, we will find the standard error:

$$SE = \frac{s}{\sqrt{n}} = \frac{40}{\sqrt{36}} = \frac{40}{6} = 6.67$$

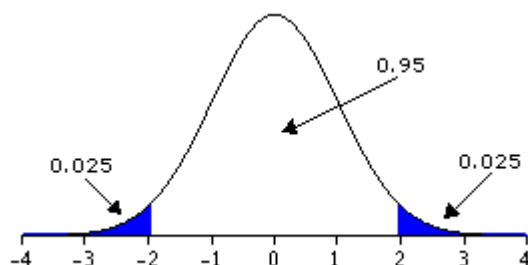
Now we can focus on the problem. The question wants us to find a 95% confidence interval as to where the mean weight of all 200,000 apples falls. To find this interval we are going to use the norm.inv() function in Excel. The norm.inv() function takes a probability, standard deviation, and mean and gives the value that will correspond with that probability.

norm.inv (probability,mean,stand_deviation)

The probability comes from the fact that we are using a 95% confidence level. Since we have a 95% confidence level we know that we will have 5% outside. We will need

to use this `norm.inv()` two times to get both values. The probabilities that we are looking for in this problem are 2.5% and 97.5%, seen in Figure 3.

Figure 3: 95% Confidence Interval [4]

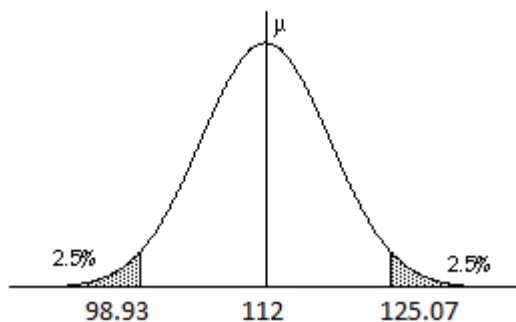


We must use these values because these values are on either end of the 95% of the data. This will have 2.5% on each tail which will add to the total 5%. Therefore, we have:

- `norm.inv(.975,112,6.67) = 125.07`
- `norm.inv(.025,112,6.67) = 98.93`

This then means the 95% confidence interval is (98.93, 125.07), shown in Figure 4.

Figure 4: 95% Confidence Interval for Example [14]



Once the confidence interval has been found, the teacher can then use Nearpod to ask the open ended question:

- What did we find?

- The true mean of the population will fall between these two values 95% of the time.
 - We are 95% confident the the true mean of the population will fall between these two values.
 - We are 95% confident that a sample of apples will have a mean that falls between these two values.
 - We are 95% confident that a sample formed in the same way will have a mean that falls between these two values.
- This is also saying that we are 95% confidence that μ is between 98.93 and 125.07.

It is important that students realize that there are times that we won't know the standard deviation of the population and a sample size of less than 30. When this happens, we have to use the student t-distribution because we are using s to approximate σ . Finding the confidence interval is exactly the same, however, we will use the $t.inv.2t()$ instead to calculate the confidence interval. The teacher will then present this problem:

- You sample 16 apples from your farms harvest of over 200,000 apples. The mean weight of the sample is 112 grams (with a 20 gram sample standard deviation). Construct a 95% confidence interval as to the actual weight of the 200,000 apples.

The teacher will then need to ask the students what makes this question different from the previous one. By doing this, the teacher can make connections as to what the students may have to do differently for this problem. The teacher will then have the students work in pairs or groups to find the standard error and the mean.

- $SE = \frac{20}{\sqrt{16}} = \frac{20}{4} = 5$

- The problem statement gives us $\mu_{\bar{x}} = 112$

We cannot use the norm.dist because we have a sample size that is under 30. Thus we need to use the student t-distribution. When we use the t.inv.2t() we will be given the t-critical value. This will tell us how many standard deviations we are away from our mean. There are 2 t.inv() in Excel. There is t.inv.rt() and t.inv.2t(). We use the 2t because we are want 2 tails in our confidence interval. The rt will only give us the right tail.

The teacher then explains what numbers are used in the t.inv.2t().

- t.inv.2t(alpha,degrees of freedom)

We find our alpha by taking 100% and subtracting our confidence interval. The teacher can then ask the students what the alpha will be for this problem.

$$\alpha = 100 - 95 = 5\% = .05$$

The students should already know how to find the degrees of freedom for a student t-distribution from a previous lesson. If they need a reminder the teacher can tell them

$$df = n - 1 = 16 - 1 = 15$$

Now that we have everything that we need, the teacher should have the students find the t-critical value and check their answer with their neighbors.

$$t.inv.2t(.05, 15) = 2.13$$

The teacher can offer another open ended problem through Nearpod to ask: What does this value tell us? They should respond by saying that it is the number of standard deviations we are away from the mean.

Now the students can find the margin of error and complete the confidence interval. The teacher will walk around as the students find the margin of error and the confidence interval, and take mental notes on what each student/group is doing. This way the teacher can call on specific groups to explain what they did to find the confidence interval. Based on the methods, the teacher can connect the different methods and help the students see what errors they made. In the end, the students should have found:

- $ME = t_{crit} * SE = 2.13 * 5 = 10.65$
- $CI = 112 \pm 10.65 = (101.34, 122.65)$

A common error that the students may make in this part of the problem is that they will not use the standard error. Instead they will use the standard deviation from the problem statement. The teacher can use this common error to help connect the meaning of standard error: that we are looking for the standard deviation of the sample distribution not of the sample.

Once the students have finished this part of the lesson, the teacher can move onto the final part of the lesson which is to give examples of where the students would see confidence intervals in real life. The first part of this is to give the students an open ended question on Nearpod: What are some examples of confidence intervals in the world around you? The answers can range:

- Political Polls
- Science studies
- Gallup Polls

The teacher can then give these examples:

Example 1: A Gallup Poll on TVs and personal wellbeing measured used the Cantril Self-Anchoring Striving Scale: “For the European data, one can say with 95% confidence that the true population for average wellbeing among those without TVs is between 4.88 and 5.26. For those with TVs the 95% confidence interval for wellbeing is much narrower – between 5.78 and 5.82 – because of the larger sample size.” [11]

Example 2 Nursing: In the study of protocol-directed sedation during mechanical ventilation implemented by nurses: “The median duration of mechanical ventilation was 55.9 hrs (95% confidence interval, 41.0-90.0 hrs) for patients managed with protocol-directed sedation and 117.0 hrs (95% confidence interval, 96.0-155.6 hrs) for patients receiving non-protocol-directed sedation.” [2]

- The teacher can ask in an open ended question: What does the confidence interval 41.0-90.0 hrs mean?
- We estimate that, for all possible patients, the median duration with protocol management is between 41 and 90 hours. We do not know where in this range the actual median duration might be, and there is a small chance that we might be wrong because it is outside these limits.

Example 3: Studies have shown that the average life expectancy of Americans is 75 years. Use the data provided in Table 3 to test this claim:

5 Analysis of Lesson Plan

To make an analysis of how well this lesson plan helped students understand the concept of confidence intervals, I gave a pre and post-assessment to the students to gauge their understanding of confidence intervals. The pre-assessment was given before the first class and the post-assessment was given a few days after the second

Table 3: Example 3 Ages

85	85	79	83
78	77	80	90
76	70	81	82
80	80	83	85
89	80	88	86
89	87	80	79
77	76	70	70
75	77	80	83
90	85	87	80
71	79	71	

class. The reason for delayed post-assessment was do to a Lab, see Appendix A, that consisted of finding confidence intervals. I wanted the students to have extra time to work with confidence intervals to better their understanding of what they mean. This is part of the connecting part of the lesson. The lab allowed students to make connections between the lecture and the examples to better understand confidence intervals.

The pre-assessment consisted of gender, age, and 3 questions regarding confidence intervals:

1. What do you think of when you hear confidence interval?
2. Describe a confidence interval in your own words.
3. What does this mean?
 - After the study was concluded, the researchers were able to say that they are 95% confident that the average age of a student about to complete college was 22 years \pm 1.5 years.

There were many answers to these questions. Of the 12 responses to the pre-assessment these were some common answers.

1. Question 1

- “How confident you are about your data interval.”
- “When I hear confidence interval, I think that it is a statistic that represents a population that is believed to be very accurate.”
- “Related to σ and how confident we are that a data point within that measurement will be relevant to what we are trying to measure.”
- “Confidence intervals are where a set of data or “findings” seems real/legit due to a sample.”

These answers are not correct but they are at least on the right track. This showed me that my students had an idea of what a confidence interval was when they heard the term. They needed clarification of where numbers come from and the interpretation of confidence intervals

2. Question 2

- “I am not sure how to describe this.”
- “An interval that is most likely to be correct, but it isn’t absolute.”
- “Seems like that numbers below a poll/data set that says “1000 students surveyed” or something to give your project legitimacy.”
- “It is how sure one is that their data is accurate to its fullest potential, including math errors.”
- “The range of numbers researchers are confident about.”
- Many answered the same as question 1.

These answers were exactly what I expected. Most of these students had not had a statistics class before. If a student had given me a correct answer, I would assume they either had read ahead in the book or had had some sort of statistics class before. Of these answers, the closest to being correct is “An interval that is most likely to be correct, but it isn’t absolute.”

3. Question 3

- “They have strong confidence.”
- “Don’t know”
- “It is most likely an accurate statistical value, but it may or may not be completely accurate.”
- “The confidence interval was 95%, so that means that due to findings, 95% of the data concludes that the average age to complete college was 22 years old, give or take 1.5 years.”
- Many others left the questions blank.

Again, these answers were not a surprise to me. Since many of the students had not had a statistics class before, they were most likely to just make a judgment based on what they already knew. Since the students already knew the 68-95-99.7 rule, the answer that states: “95% of the data concludes that the average age to complete college was 22 years old, give or take 1.5 years” is an accurate answer for the knowledge that they have.

At the end of the lesson and the lab, I presented them with a post-assessment. The post-assessment had gender, age, and four other questions. Two of the questions were ones they had seen before. The third question was changed from an open ended question to a multiple choice question. This was to needed to gather data about the success of the lesson. The fourth was a rating.

1. What do you think of when you hear confidence interval?
2. Describe a confidence interval in your own words?
3. After the study was concluded, the researchers were able to say that they were 95% confident that the average age of a student to complete college was 22 years \pm 1.5 years. Which of the following do you agree with?

- There is a 95% chance that the age of a student that graduates college will be between 20.5 and 23.5.
- There is a 95% chance that the average age of all graduating students will fall in the given range.
- There is a 5% chance that a student picked at random would have an age that does not fall in the range.
- There is a 5% chance that the average age of all graduating students will not fall in the given range.
- If I take another sample of n graduates, there is a 95% chance that my confidence interval will capture the population mean.

4. How confident are you that you could compute and understand a confidence interval? Scale 1 2 3 4 5

There were more students in class the day that I gave the post-assessment. This means that I had 18 responses instead of 12. The responses to these questions were much better than the pre-assessment.

1. Question 1

- "An expected range of numbers that we are not 100% sure of accuracy."
- "A way that we can draw conclusions about the means of set(s) of data."
- "I think of a range of confidence, usually about the statistical mean. For example, I think of something like $2.5 \leq \mu \leq 5.7$."
- "How confident we are that a number falls between a certain range."

These answers are very common ones indicating what students think about when they hear the term confidence interval. As we can see, there is a consensus that confidence intervals are a range of numbers that say something about a mean or

proportion. When reflecting on this question, as an educator, it would be better to have more responses like: “I think of a range of confidence, usually about the statistical mean. For example, I think of something like $2.4 \leq \mu \leq 5.7$.” While there are still some issues with this answer, it gives a very brief description of a confidence interval. If every student that leaves a high school statistics class can have at least this basic understanding, they will be in a good place to make inferences about reports they encounter in their daily lives.

2. Question 2

- “A confidence interval, for example a 95% confidence interval, tells us that we can say with 95% confidence that if we were to take many samples and generate many confidence intervals, the mean of the sets of data would be between our range.”
- “A confidence interval is a prediction of what a mean is going to be based on stats. The prediction is a range.”
- “The percentage of confidence that the actual mean of a sample is in between two values.”
- “The confidence that a mean will fall between a high and low of a numeric value.”

These answers also gives a good understanding of what topics need additional review time or require more instruction time. One confusion that would need to be discussed is the difference between a confidence level and a confidence interval. When we look at the answer: “The percentage of confidence that the actual mean of a sample is in between two values,” we can see that there is confusion on what is actually in the confidence interval. Therefore, it would be helpful to review that it is actually the true mean of the population that falls in this interval not just the mean of the sample.

3. Question 3

- The correct answers were choices 2, 4, and 5.
- There were only a few students that actually gave all three answers.
- The majority of student chose 2 and/or 4.

This question shows that there is a consensus of the interpretation of confidence intervals. Figure 5 shows the distribution of the responses to this question. With

Figure 5: Responses to Question 3



the knowledge gained from this question, it is clear that there is a good amount of interpreting, questioning, and reasoning in the lesson. However, there is always room for improvement. The more reasoning that the students are able to do, the better they will be at interpreting confidence intervals in their daily lives.

One part of teaching is reflecting on how a lesson went and seeing how we can improve. There are some things that would need to be changed after the completion of the lesson. This lesson was only meant to be a two day lesson. However, it seems from the responses that there are still some topics that need to be covered in more depth. This means that we should take more time on confidence intervals. Another

change that I would make to this lesson would be to include some sort of project or homework assignment that requires students to go home and look for confidence intervals around their daily lives and report on them. With this project the students will be able to have a better understanding of the reasons for learning confidence intervals. If the students do not see the reason for learning confidence intervals, then they will not be as engaged in the classroom. Another important part would be to be sure to change this lesson's examples to fit the class. The students will have specific likes and dislikes. If the students are engaged in the topics, then they will be more likely to understand what it is that they are learning and hopefully retain more information.

6 Conclusion

In high school statistics there are many concepts that a student will cover. Among these concepts is confidence intervals. In order to be an informed member of society it is important that students understand confidence intervals. Whether a student goes on to college or enters the work force, they will encounter information about confidence intervals everywhere.

Whether a person is studying a science or a math or just living in a community, they will encounter statistics and, in particular, they will encounter confidence intervals. For those who are going on into the college field, they may read multiple studies that are testing the same thing. If they do not understand what a confidence interval is they will not be able to make judgments as to whether these studies are statistically sound or not. If a person living in a town sees a confidence interval for a poll about their town's opinion, they need to understand where the pollsters got the information and what the confidence interval means.

In this thesis we have discussed that the confidence interval is in the Common

Core State Standards. Even though the standards do not specifically state the name confidence interval, it is implied. The standards address about being able to make judgments based on data. The standards address about how confidence intervals provide a way to support teaching students about constructing viable arguments. They require that a student be able to make estimates about a population mean or proportion and to evaluate reports on data. By using a confidence interval, students will be able to do this. Even the math practice standards have confidence intervals in them. They talk about being able to “[c]onstruct viable arguments and critique the reasoning of others” [13]. A knowledge about confidence intervals give the students the ability to make these judgments of other peoples’ statistics.

Finally, I have presented a possible lesson that could be used to teach confidence intervals to a high school statistics class. The lesson shows the students how to use the 68-95-99.7 rule to find a confidence interval as well as finding a margin of error. It also shows the students how to use technology to find the confidence interval needed. In the analysis of the lesson, there were signs of things that would need to be changed. This is part of teaching. When a teacher presents a lesson, it is important that he or she goes over the lesson to see if there is anything that they would change for next time. In this lesson, I would definitely add some sort of project or homework that requires students to find confidence intervals in their own lives. If they find them in their daily lives, they will hopefully be more engaged in the topic. Another big change would be to find examples that fit your class. By the time confidence intervals are taught in a high school stats curriculum, the teacher should know his or her students well enough to find examples that are better for their students to keep them engaged.

It is the teacher’s role in a classroom to make sure that the students are engaged in meaningful mathematical discussions. If the teacher just lectures and does not let the students explore the concepts, the students will not be engaged. By using meaningful mathematical discussions about topics that are interesting to the students, the teacher

will hopefully keep the students interested in the topic. If the students are engaged, they will hopefully retain the information that they are learning and understand why it is important.

Confidence intervals are everywhere in life. It is important that we prepare our students to enter into the world of statistics so that they can be ready to make judgments on their own, and be sure that they can support those judgments.

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A

Lab on Confidence Intervals

A.1 MA 207 - Lab 8[3]

In this lab, we will discover and examine confidence intervals, along with the associated interpretations of such intervals. This lab will ask you to calculate values in Excel, and, in some cases, interpret the results of your calculations. Remember, it is essential to write all of your answers in complete sentences using appropriate grammar, punctuation, and terminology. Submit your lab write-up in a Microsoft Word (or compatible) format and the Microsoft Excel spreadsheet containing your calculations through Moodle. You may work on your own or in groups of 2 on this assignment. If you work in a pair, be sure that you have time to meet together outside class to finish up this lab, as you may not be able to complete it within class time.

Only one student needs to submit a solution, but both names should be included. Refer to the Instructions for Lab Write -ups document on Moodle for information on how your lab should be formatted.

All calculations should be done in the Excel file corresponding to Lab 8 found on Moodle. This spreadsheet will have locations to place all of the necessary results; fill in all of the values (e.g. Mean, Standard Deviations, z-score (or t-score), Margin of Error, Upper and Lower Bounds of Confidence Interval, etc.) to receive full credit.

A.2 Confidence Intervals-Example

For this example, we ask that you follow along in the Excel data. We will be finding a 90% confidence interval for the data labeled Heart Rates in the Lab 8 Excel data.

First of all, we have to determine the situation we are dealing with. There are two big questions: Should we use the normal distribution or the t-distribution and

what sort of mean are we finding (i.e. sampling distribution, difference of means, or proportion)?

In this situation, we have sampled 130 random individuals and determined their heartrate in bpm (beats per minute). Therefore, we will use a sampling distribution for our confidence interval.

Now, we use the normal distribution when our sample size is greater than 30 or if we know the population standard deviation. When our sample size is less than 30, we use the t-distribution. In this case, our sample size is 130 so we will be using the normal distribution. With that taken care of, we now find the mean and standard error of our sample. Complete all calculations in your Excel spreadsheet. In this case, our mean is 73.762 and our standard deviation is 7.062. We then find our standard error by dividing our standard deviation by the square root of our sample size. This gives us a standard error of 0.619.

Before we can find our margin of error, we need a z-score or t-score that corresponds to our confidence level. Since we are using the normal distribution, we will be using a z-score but we will show you how to find both.

z-score: For a z-score, we use the following command:

$$= \text{NORM.S.INV}(\text{confidence} + \frac{1 - \text{confidence}}{2})$$

This may seem confusing but remember, we have to account for the lower tail when finding the z-score. An easy way to think about this is to take norm.s.inv of halfway between our confidence and 1. For example, for our 90% confidence interval, our z-score would be given by $= \text{NORM.S.INV}(0.90 + \frac{0.90}{2})$ or $= \text{NORM.S.INV}(0.95)$.

t-score: For a t-score, we can use a similar command:

$$= \text{T.INV}(\text{confidence} + \frac{1 - \text{confidence}}{2}, \text{degrees of freedom})$$

Remember that the degrees of freedom is given by one less than our sample size. We can also use:

$$= \text{T.INV.2T}(1 - \text{confidence}, \text{degrees of freedom})$$

which is a bit more compact.

We can now find our margin of error by multiplying our z-score and our standard error together. Our margin of error is 1.019.

Finally, our confidence interval is given by (mean-margin of error, mean + margin of error). In this example, our confidence interval is (72.743, 74.780).

We can do basic hypothesis testing with this now! Suppose someone claimed that the true population mean for heart rates was 70. We can reject this with 90% confidence because 70 is not in our confidence interval.

Suppose that someone claimed that the true population mean for heart rates was 73. This number is in our confidence interval so we would fail to reject them, or we would believe them.

Last things to note:

- A confidence interval only tells us about the population mean, it doesn't tell us about any specific sample (heart rate in this case).
- If we construct a 90% confidence interval, then it is incorrect to say that there is a 90% chance that the true population mean lies in that confidence interval. It is correct to say that, if we took many different samples and generated many confidence intervals, we would expect the true population mean to be in 90% of those confidence intervals. Interpret the confidence intervals that you will create in the future in this correct manner.

A.3 Confidence Intervals-Your Turn

A.3.1 Body Temperatures

For this section, open up the tab labeled Body Temperatures in the Excel spreadsheet. We want to find a 95% confidence interval of the population mean of body temperatures using this sample of 29 individuals. Calculate the necessary values (e.g. Mean, Sample Standard Deviation, Sampling Distribution Standard Deviation, t or z-score, Margin of Error) to find your confidence interval.

1. Now, give and interpret your confidence interval in a sentence or two: what does this interval tell us about the population mean of body temperatures?

A.3.2 Blood Pressures

For this section, open up the tab labeled Blood Pressures in the Excel spreadsheet. This data describes the increase of blood pressure after an individual takes a certain medication. We want to find a 99% confidence interval of the population mean of the increase in blood pressure using this sample of 7 individuals. Remember the importance of this small sample size (will we use a z-score or a t-score?). Calculate the necessary values (e.g. Mean, Sample Standard Deviation, Sampling Distribution Standard Deviation, t or z-score, Margin of Error) to find your confidence interval.

2. Now, give and interpret your confidence interval in a sentence or two: what does this interval tell us about the population mean of the increase in blood pressure? The company that produces the medication claims that it does not increase blood pressure; is this a valid claim (i.e. does the confidence interval include the value that corresponds to no increase in blood pressure)?

A.3.3 Proportions of Voters

For this section, open up the tab labeled Proportions of Voters in the Excel spreadsheet. This data describes the proportions of voters that said they were voting for candidate A in an upcoming election. There are two samples on this tab: one of sample size 50 and one of sample size 500; the proportion of voters saying they will vote for candidate A remains the same: 62%. We want to find 95% confidence intervals of the population mean of the proportion of individuals voting for candidate A. Calculate the necessary values (e.g. Mean, Sampling Distribution Standard Deviation, z-score, Margin of Error) to find your confidence intervals.

3. Imagine that the survey has only been conducted on Sample 1 (50 individuals); should the other candidate, candidate B, stay in the election? Use your confidence interval for Sample 1 to explain; be sure to include the confidence interval you calculated.
4. Imagine that the survey has only been conducted on Sample 2 (500 individuals); should the other candidate, candidate B, stay in the election? Use your confidence interval for Sample 2 to explain; be sure to include the confidence interval you calculated.
5. What do the results from questions 4 and 5 tell us about the effects of larger sample sizes on confidence intervals?