Mathematics and Music:
Integrating the World Wide Web into Today's Education System

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April 16, 1999

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This thesis for honors recognition has been approved for the Department of Mathematics, Engineering, Physics, and Computer Science, Carroll College, Helena, Montana.

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* A compact disc has been enclosed as an appendix to this thesis. The contents of the web page we created are stored on this compact disc, so the reader can view the web page by using a computer with a CD-ROM drive.

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Acknowledgements

We would like to take this time now to thank the people who have encouraged us to remain persistent and complete this project. To Marie Vanisko who is our Thesis Director and who invited us to participate in the web conference in the first place, to Dr. John Scharf ---our reader and a great help through out, and to Dr. Roderick Thronson who is also our reader, thank you all. We would also like to thank the Math, Physics, Engineering, and Computer Science Department and all of the professors that helped us, especially Dr. Anthony Szpilka and former visiting professor Frank Wattenberg. These two helped us greatly with the web page labs.

We can not forget those people who helped us with our web project without whom we wouldn't have had such an interesting topic to write about. These people are Amanda Abbott, web participant, and Christopher May, web page advisor. Amanda and Christopher played an integral part in developing the web page. Thank you both so very much; we appreciate it. And finally we would like to thank our families and friends who helped us to not give up when we got frustrated and to put up with us through the course of the project. They are the ones who helped us get where we are today. Thank you all.

Abstract

In the global village, technology continually grows, enabling people to communicate, solve problems, and convey ideas. Students can benefit by using technological advances to enhance their education. The ultimate goal of our thesis is to encourage the use of technology in the classroom by introducing teachers and students to the World Wide Web. Specifically, we focus on using computers and other technology to enhance mathematics education at the high school level. To meet our goal, we created a web page entitled, "Mathematics in Music: Integrating Education and the World Wide Web." We offer the web page as an example of a learning tool, which will motivate students to learn about mathematics while gaining computer skills. For teachers, pages like ours will act as a convenient alternative to hours spent digging through files and books in order to find creative lesson plans. Our web page includes four main sections: an overview of mathematics and music relationships, a section about harmonies and frequencies, another on beats and resonance, and a section of hot links connecting to other related sites. Aside from the web page itself, we have written an explanation of the mathematics and lab procedures used throughout the web page. Information concerning how to use the page in a classroom to boost students' learning is also included. Finally, tips and references are offered to teachers, should they wish to create their own web pages.

Introduction

While Mrs. Wagner writes trigonometry identities on the dry-erase board, Jason mischievously flicks eraser chunks across the room, pegging Chris in the temple. Dustin and Lorie whisper back and forth, placing bets on whether or not the girls' basketball team will take the state title, and Jami writes a note to Jen, who is sitting three seats behind her. Meanwhile, Anna tries desperately to stay awake, as boredom tugs at her eyelids. Mrs. Wagner, brilliantly cognizant of every move the class makes, sighs to herself and tries to think of a more creative way to present the remainder of the lesson.

Mrs. Wagner faces many of the same challenges high school teachers across the nation face each day. How can teachers motivate kids to learn? How can teachers make learning meaningful to students? Although we realize students themselves are the only ones who have ultimate control over their learning, we feel that teachers can encourage learning by using a variety of techniques. One such technique requires integrating technology into the classroom and applying it to topics which students find naturally interesting.

Technology is already being used as a resource to provide teachers with another medium to present information to students. As future educators, we need to realize the importance of technology and students' exposure to it and integrate technology into the curriculum. With the use of technology, some of Mrs. Wagner's daily struggles may be put to rest, although being an educator will always present her with new challenges.

This project challenged us to find creative methods of integrating technology, namely the World Wide Web, into a high school curriculum. To accomplish this task, we selected a topic of interest and developed a web page to be used by teachers and students.

Through the use of our page, we hoped to convey the classroom effectiveness of the Internet and other technologies, such as the Calculator-Based Laboratory (CBL), the TI-82 and TI-92 calculators, and computer software programs like *Mathematica*.

The topic selected addressed the connections between mathematics and music. We chose this topic because of our exposure to music and our knowledge of the relationship between mathematics and music. We also reflected back on our high school days and recalled our own interest in music and how many of our classmates shared similar interests. Knowing that music is a common interest shared among many students, we hoped to appeal to them and attract students by providing projects, labs, *Mathematica* programs, and general information. This would enable them to see how mathematics can relate to anyone's genuine love of music.

We decided to present this information using Disney graphics to aesthetically enhance the web page with the hope of drawing teachers and students to the information. The web project was designed to be user-friendly so that any user with any level of exposure to the Internet would be able to move throughout our site, allowing the users to focus on the information presented while becoming comfortable with the Internet.

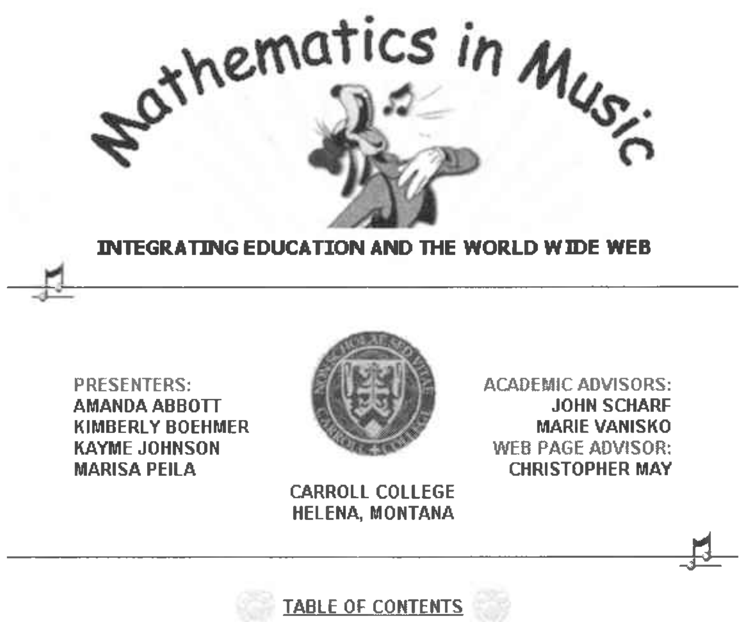
The purpose of the web page was to provide teachers with some easy-to-use, motivational projects and labs, which they could integrate into the curriculum. Through these activities we hoped to motivate the students to learn mathematics while exploring the Internet and showing them applications of math in today's world. These lesson plans basically provide a different approach to teaching math skills, and students seem to find the use of technology very interesting.

While compiling the necessary information to construct the web page, our creativity level in the classroom blossomed. Creativity is an important skill for teachers to take into the classroom because it can motivate students to learn. This project has taught us how to present information at a high school level, which can be difficult for us college students.

Through the progression of this project, we will explore our web page and explain how it may be integrated into a classroom effectively. We will also look at technology's role in conveying this information to the student.

Chapter 1: Web Page Format

Relationships existing between math and music are as vast as the Montana sky, thus we explore only a few of these relationships, such as harmony, frequency, beats, and resonance, in our web page. When creating a web page, one must consider how to organize the information. Therefore, we decided to break our page into sections that linked to one another in a logical fashion. Upon arriving at the Mathematics and Music web page, a visitor first encounters a colorful title page, which is illustrated below:



Incidentally, the title page includes a musical sound byte of the famous Mickey Mouse song. When creating a web page, remember that sound can add an extra element of fun to the project. From the title page, visitors click on a link to reach the table of contents, which lists four main sections of the page. The first is entitled, *The*

Relationship Between Mathematics and Music. This section offers a user-friendly introduction to the mathematical and musical relationships discussed throughout the web page. Any teacher wanting to create his or her own web page to teach others could begin the page with a format similar to the one we chose. The text on the following page comprises the content of our web page overview.

The Relationship Between Mathematics and Music

Overview and purpose of web page

Mathematics and music tie together like a tightly bound shoelace. First, math and music share one very uncommon property--they are both universal languages of the world. A young student in Russia and an old musician in Canada could play a piece of music together, and each would understand the other's interpretation, regardless of their native verbal languages. Similarly, a child in Belgium could add the cost of groceries just as easily as an American child could tally his total purchase at the candy store. Truly, any person could relate to another when speaking the universal languages of mathematics and music.

Secondly, the scientific relationship represents another powerful tie which bounds math and music. Physics plays a great part in the music we hear. Behind the melodious, free-spirited, and ear-pleasing tunes we hear, lie the surprisingly scientific and mathematical concepts which make music possible. For instance, the harmony we hear in chords played on the piano is possible because of sound waves' physical properties. The range, or pitch, of a note can be calculated using frequencies of waves. These physical properties of music can often be analyzed mathematically. Some fundamentals of music link directly to math. For example, musicians use math specifically when they form rhythms based on time measurements, with some rhythmic patterns lasting only fractions of a second.

As one can see, music and math can be viewed from countless interesting and scientific perspectives. When first developing our web page, we sought to create a highly motivational site for both teachers and students to learn more about mathematics and its relationship to music. As you browse through our pages, you will notice several links to other sites on the World Wide Web. These links provide additional information, as well as fun facts about mathematics and music.

Within this page, we offer labs and brief descriptions, which teachers may want to use to illustrate concepts to their students. Included in the labs are programming codes for *Mathematica* (a math software program), and both the TI-82 and TI-92 calculators. These codes may be downloaded for educational use. Also, our web page is "student-friendly," so any student who wishes may walk through our page and discover new angles from which they can view music.

By reading the overview excerpt, the viewer can see that our goal was to offer a friendly welcome to teachers or students who visit the page and to pique the interest of any learner. The overview of Mathematics and Music invites learners to stroll through the information and view the various labs that are available.

Another component of the *Mathematics and Music* web site is a section entitled *Harmonies and Frequencies*. This section provides the browser with an open door to the mathematics behind the beautiful sounds created by musicians and their instruments. Let us open that door ourselves…

Chapter 2: Harmonies and Frequencies

We begin this section with a brief overview explaining harmony and frequency (See CD Appendix). We also introduce a lab which high school students and teachers could use as a learning tool.

Showing how music is created though a series of sound wave patterns is the goal of the *Harmonies and Frequencies* section. Once learners understand this concept, they are introduced to a special mathematical concept called the Fourier series. This series shows how any complex curve, such as that created by a musical note, can be regenerated through the summation of sine and cosine curves.

According to Larry C. Andrews, author of *Introduction to Differential Equations with Boundary Value Problems*, the Fourier series was first studied by the French mathematician J. B. J. Fourier in an effort to simplify the solving of heat conduction problems [**1**]. His work has many other useful applications, as well. We use the Fourier series to describe periodic motion. When presenting periodic motion to students, a teacher could list several examples of it. For instance, a sound wave produced by a musical instrument is an example of a periodic function. Other examples include the harmonic motion of man-made structures like bridges, the motion of a spring, and ocean waves.

High school students with a trigonometry background should recognize that two periodic functions are sine and cosine. Therefore, a teacher can make a smooth transition from discussing sine and cosine into the Fourier series, which is a special sum of cosine and sine curves. When presenting the Fourier series to a high school calculus class, the concept of integrals could be familiar to them, thus they could practice applications of the

Fourier series. However, most high school curriculums do not include an in-depth study of the Fourier series. Mathematicians use the Fourier series (listed below) to approximate periodic functions over a set interval from *-p* to *p*.

Fourier Series

$$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos\frac{n\pi t}{p} + b_n \sin\frac{n\pi t}{p})$$

where n = 1,2,3… **(1)**

"What do the constants, a_0, a_n, and b_n, represent in this series?" one might ask. Their equations are:

$$a_0 = \frac{1}{2 \cdot p}\int_{-p}^{p} f(t)dt \qquad \textbf{(2)}$$

$$a_n = \frac{1}{p}\int_{-p}^{p} f(t)\cos\frac{n \cdot \pi \cdot t}{p}dt \qquad \text{for n = 1,2,3…} \qquad \textbf{(3)}$$

$$b_n = \frac{1}{p}\int_{-p}^{p} f(t)\sin\frac{n \cdot \pi \cdot t}{p}dt \qquad \text{for n = 1,2,3…} \qquad \textbf{(4)}$$

In the coefficient equations, *n* represents the number of terms out to which the mathematician chooses to extend the Fourier series, and *p* refers to the interval over which one is approximating. Looking at the formulas, note that the calculation of the coefficients, a_0, a_n, & b_n requires the use of integration, which is not normally introduced to students until they are enrolled in a calculus class. However, the basic concept of the Fourier series---the focus of using pieces of sine and cosine curves to match a complex curve---can be appreciated at many different high school levels. Therefore, we suggest using our introduction to the Fourier series and the related lab as a supplement or enriching addition to a lesson explaining a variety of "matching functions" such as the Fourier series and Taylor Series, etc.

The lab concerning the Fourier series in the *Harmonies and Frequencies* section, will enhance any lesson used to show the relationship between mathematics and music. The lab illustrates how the Fourier series can approximate different functions, in particular, sound wave curves. The web page design enables teachers and students to either view a sample lab write-up or recreate and complete the lab themselves. We have included the necessary *Mathematica* code within our web page for all the available labs, and the code can be either viewed or downloaded for educational use.

The Harmonies and Frequencies Sound Lab contains a detailed procedure. To actually follow the lab procedure, the materials needed include a Calculator-Based Laboratory (CBL), a sound probe, a calculator (TI-92 or TI-82) and several musical instruments. For readers who are unfamiliar with these technologies, we will briefly explain their uses. The CBL, TI-82, and TI-92 are products manufactured by Texas Instruments, and their use in the classroom continues to grow. The CBL is a data-collection system having its own microprocessor and memory, and students can use it to measure motion, temperature, light pH, force, sound, and other characteristics with an adapter and different probes [**3**]. Once the CBL collects the data, students can view and analyze the data by hooking the CBL up to a variety of Texas Instruments calculators, such as the TI-82, TI-92, and TI-83. Incidentally, labs within our web site use the TI-82 and TI-92 calculators; however, the TI-83 can easily substitute the other calculators in the lab procedures.

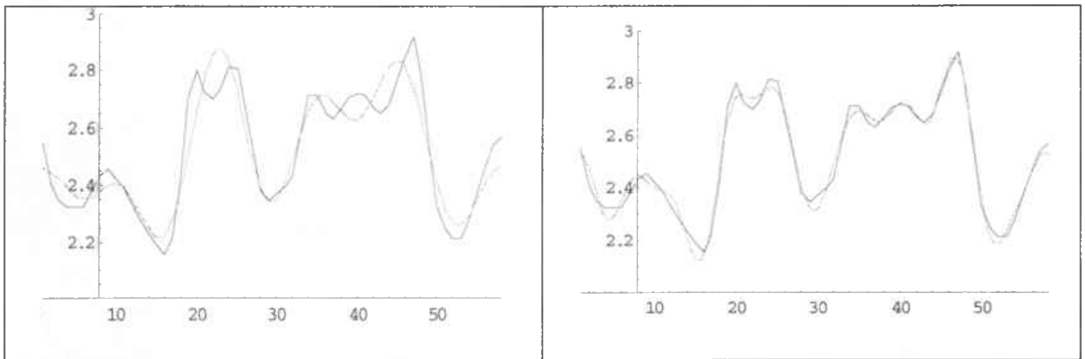
In this lab, the CBL is used to format the musical instrument's sound into a language the calculator can understand. The musical note is transferred to the calculator via the CBL. When the calculator receives the wave data, it lists the data in the form of coordinate points. From there, the student is able to graph the coordinate points on the calculator and actually view the curve of the musical instrument's note. The TI-92 is capable of saving the list of data points and the related graph. The TI-82, on the other hand, cannot save the data points. Thus, teachers only having access to TI-82 calculators will have to require students to complete one trial at a time, keeping in mind that the actual data points will be erased once students input new data from another trial into the calculator. After the information is listed in either calculator model, students may copy the data points into a software program like *Mathematica* and begin analysis using the Fourier series. *Mathematica* was created by Wolfram Research, Incorporated, and has tremendous capability in manipulating data. In his book, *Mathematica: A System for Doing Mathematics by Computer*, the company's founder Stephen Wolfram describes *Mathematica* as a software system that can perform *numerical, symbolic*, and *graphical* computations [**8**].

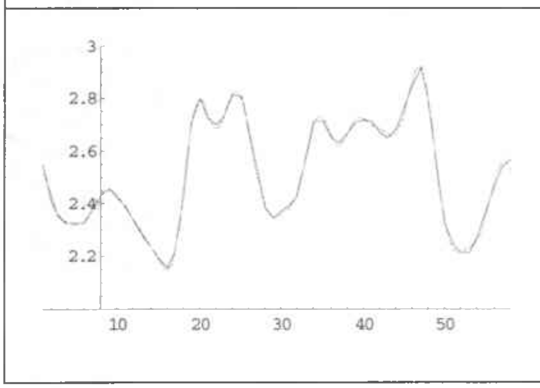
During the lab, students form cooperative learning groups and complete the lab together. The lab requires the playing of two instruments, a clarinet and a piano; one could substitute these instruments with any others if he or she desires. Students play the same note in three different trials, and the sound wave is recorded onto the calculator via the CBL. Specifically, when we completed the lab ourselves, an **E** was played on the clarinet, and a **D** was played on the piano. Both notes were lower than middle **C**. Incidentally, the notes **E** and **D** are essentially the same note, or frequency, due to the natural keys in which the clarinet and piano are played.

Using notes with relatively low frequencies enables students to see a single period more clearly on the graph. Should they use a note with a higher frequency, students will find the wavelengths to be much smaller, making it difficult to analyze one period. Once the notes are recorded among student groups, the data can be transferred to *Mathematica*. The data consists of the points listed on the calculator which comprise the sound wave that the instrument produced. Now that the data has been captured by *Mathematica*, it can perform exhausting computations, which will save students great amounts of time (See CD Appendix).

Within the *Mathematica* code, which will be detailed below, a few items must be defined for the software to evaluate the data. The value k indicates the number of sine and cosine terms which will be summed in the series. As k increases, the Fourier series will more closely approximate the actual sound wave function, or note, that was created through the instrument. Throughout the lab procedure, the teacher will want to encourage students to observe the changes which occur when they alter the k value. We suggest having students first try a smaller value for k, such as 1, and then progress to assigning larger values to k, such as 30. Students will learn a great deal by analyzing the graphs because the illustrations of the sound curves act as a great visual tool for describing the usefulness of the Fourier series. Figure 1 shows the Fourier series approximation using three different values for k.

Figure 1





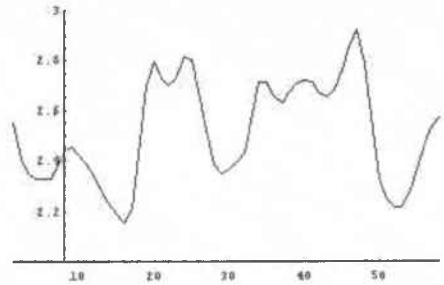
Upper Left: Fourier Series approximation of **Clarinet** trial with *k=5*
Upper Right: Fourier Series approximation of **Clarinet** trial with *k=10*
Lower Left: Fourier Series approximation of Clarinet trial with *k=15*

In addition, *Lambda* must also be defined. *Lambda* represents one period of the sound wave, which is also the interval of discrete points over which the Fourier series will be applied in the trials. For classes other than calculus, we recommend having students use the lab as a way to visualize what happens as *k* increases, rather than focus too heavily on the complexity of the Fourier Series. Figure 2 on the following page gives an excerpt of the *Mathematica* code used in the **Clarinet** trial.

Figure 2

Clarinet

```
Clarinet = {2.44049, 2.35653, 2.32854, 2.32295, 2.32854, 2.37892, 2.429
Period = Table[Clarinet[[c]], {c, 124, 181}];
dp1 = ListPlot[Period, PlotRange → {{1, 58}, {2, 3}}, PlotJoined → True]
```



- Graphics -

```
Clear[Lambda, a, b, P, n, t, k]
k := 30
Lambda := 57
```

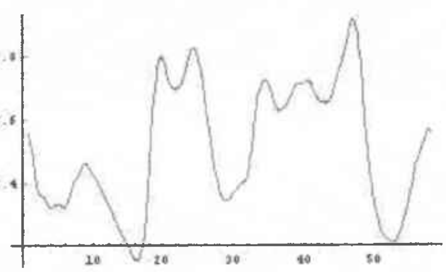
$$a[0] = \frac{\sum_{t=1}^{Lambda} Period[[t]]}{Lambda};$$

$$a[n\_] := a[n] = \frac{2 \sum_{t=1}^{Lambda} Period[[t]]\, N\left[Cos\left[\frac{2\pi n t}{Lambda}\right]\right]}{Lambda}$$

$$b[n\_] := b[n] = \frac{2 \sum_{t=1}^{Lambda} Period[[t]]\, N\left[Sin\left[\frac{2\pi n t}{Lambda}\right]\right]}{Lambda}$$

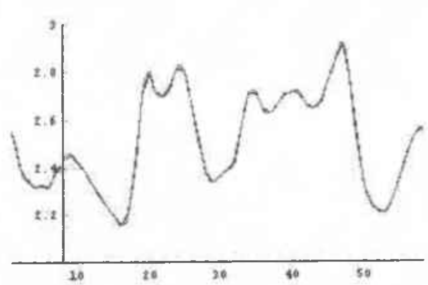
$$P[t\_] := a[0] + \sum_{n=1}^{k}\left(a[n]\, Cos\left[\frac{2\pi n t}{Lambda}\right] + b[n]\, Sin\left[\frac{2\pi n t}{Lambda}\right]\right)$$

```
fp = Plot[N[P[t]], {t, 1, 58}, PlotStyle → {RGBColor[1, 0, 0]}]
```



- Graphics -

```
Show[dp1, fp]
```



- Graphics -

When viewing the code, notice that a[0], a[n], and b[n] represent the constants found within the Fourier series. P[t_] represents a form of the Fourier series. You will notice that the Fourier series is written in a slightly different form than the notation we presented earlier. Many sources present the Fourier series in a different manner, but keep in mind that authors and mathematicians are trying to illustrate the same concept. In *Mathematica*, the code lists the Fourier series in the form:

$$a[0] + a[1]Cos\frac{2\pi \cdot 1t}{Lambda} + b[1]Sin\frac{2\pi \cdot 1t}{Lambda} + a[2]Cos\frac{2\pi \cdot 2t}{Lambda} + b[2]Sin\frac{2\pi \cdot 2t}{Lambda} + ... + a[n]Cos\frac{2\pi \cdot nt}{Lambda} + b[n]Sin\frac{2\pi \cdot nt}{Lambda}$$

(5)

Compare that to:

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos\frac{n \cdot \pi \cdot t}{p} + b_n \sin\frac{n \cdot \pi \cdot t}{p})$$

(6)

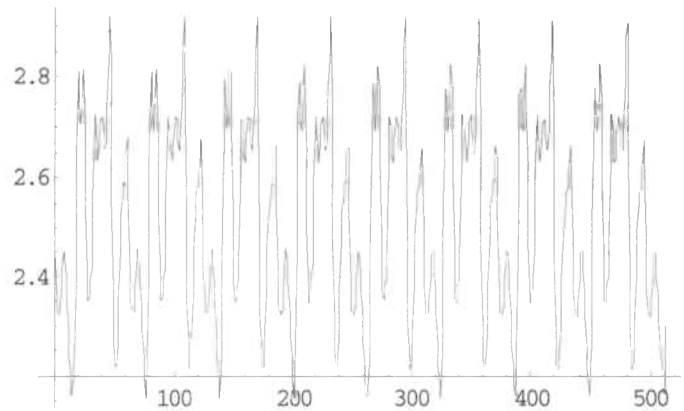
Why the discrepancy? The difference lies in how each notation represents the interval over which the Fourier Series is approximating a function. In *Mathematica*, the software system was not given a function to work with, but rather a set of *discrete* data points which were gathered from the calculator during the lab. Although the graph looks continuous on *Mathematica*, it is actually a large set of points that are connected. Therefore, the coefficients a_n and b_n are found by summing the areas between each data point using a type of numerical integration. The *Mathematica* code parallels the concept of using Riemann sums, whereas in the regular form of the Fourier series, coefficients are found by taking the antiderivative of the function which is being approximated. Calculating the antiderivative is possible because when given an actual function to work with, a mathematician can approximate that function over a *continuous* interval. For instance, in the common form of the Fourier series, coefficients are found by integrating over the interval from -*p* to *p* in equation **3**: $a_n = \frac{1}{p}\int_{-p}^{p} f(t)\cos\frac{n\cdot\pi\cdot t}{p}\,dt$.

The *Mathematica* code, on the other hand, approximates the curve over a set interval of data points, also called *Lambda*. *Mathematica* requires numerical integration in calculating a_n and b_n because it has no *f(t)* function over which to take an antiderivative. Ideally, mathematicians would prefer to approximate a curve over a continuous interval to be as accurate as possible. The lab trials, however, involve gathering raw data points, so numerical integration would be appropriate to use.

Looking at the *Mathematica* code, the reader can see that data point sets of our three trials of the clarinet note, **E**, were named **Clarinet, Clarinet 1**, and **Clarinet 2**. Incidentally, the programming language in *Mathematica* allows users to name pieces of information anything they choose. Students could name their data sets or approximating functions anything they want. Once certain pieces of data are named, students can refer to that data by using its given name, rather than retyping the original data.

From each of our named data sets we extracted the graph for one period of the sound wave. In order to graph the data, we created a table of the data points and limited it to a certain range. Before students are able to graph one period of the sound wave, however, they will want to graph the entire set of data points in order to observe the periodic motion of the sound wave. Then students will be able to pick out where one period lies and try to graph that period of the overall sound curve. A graph of the entire set of data points for the **Clarinet** trial is displayed in Figure 3.

Figure 3



In viewing the *Mathematica* code in Figure 2, one can see that for the **Clarinet** trial, a full period could be graphed by generating a table of the 124^{th} point listed in the original data set named **Clarinet** to the 181^{st} point. These 57 data points are the basis for defining *Lambda* to be 57. Fittingly, we named this table **Period** because in the trials shown, the range was assigned a value equal to approximately one period of the sound curve. These specific points just happen to be one interval showing a full period, but students could choose different points to represent a period of the sound curve. Using the **ListPlot** command enabled *Mathematica* to graph **Period**. The graph of each **Period** table was named, as well. For instance, the graph of the data points for one period of the **Clarinet** trial was given the arbitrary name, **dp1**. Next, the Fourier series approximation of the actual data points was graphed and named. The **Clarinet** trial's Fourier series approximation was named **fp**.

Finally, the graphs of the actual data points and the Fourier series curve were plotted over one another by using the **Show** command in *Mathematica*. Pointing to the

Clarinet trial as an example once again, the reader will notice the code for the overlay of these graphs is:

Show[dp1, fp]

Students will easily see how closely the Fourier series can approximate a curve comprised of actual data points by looking at the actual and approximated curves on top of one another. Students should also note that the black curve represents the actual data points of the sound wave, while the colored curve represents the Fourier series approximation (See CD Appendix). They will recognize error in the approximation if they see any gaps between the actual curve and the Fourier series approximation curve. Students can use the same process followed for the clarinet trials to analyze the piano trials. After completing the *Harmonies and Frequencies* section of the web page, students have the option to move on to the *Beats and Resonance* section to expand their learning.

Chapter 3: Beats and Resonance

We created the *Beats and Resonance* section to help teachers incorporate a math topic, like trigonometric identities, into an art topic such as music. Our section gives the teacher a resource for this kind of interdisciplinary activity. We have several goals for students who explore our web project. We want the students to see that what they are learning really *does* have an application in another field of study and that it just isn't "math" that they are learning. From the examples and equations we provide, we hope students can create graphs of their own and explore the concepts of beats and resonance in different ways. We want students to look at what we have given them and expand their thinking by observing what happens when they change a number here and there. Keeping these goals in mind, we designed the *Beats and Resonance* section of our web page, and we will now explain this section.

When clicking with the mouse on the page titled *Beats and Resonance*, a viewer is directed to the overview page. Similar to the *Overview of Harmonies and Frequencies*, this page gives a brief scenario of a classroom situation where one might use the study of beats and resonance (See CD Appendix). We then discuss the phenomena of beats and resonance and explain why and how people use them. This page, like all of our overview pages, was designed to be a brief information page to catch the attention of the reader and motivate him or her to continue reading through the project.

From the overview page viewers are directed to the *Beats and Resonance Mathematica Lab* page. This page gets the students thinking by incorporating their knowledge of trigonometric identities and explaining how these identities are used in modeling and interpreting musical instruments.

According to William E. Boyce and Richard C. Diprima, in their book *Elementary Differential Equations and Boundary Value Problems*, the general trigonometric equation for beats is:

$$u = c_1 Cos(w_0 t) + c_2 Sin(w_0 t) + \frac{F_0}{m(w_0{}^2 - w^2)} Cos(wt)$$

(7)

[2]

In this equation, c_1 and c_2 represent amplitudes of the cosine and sine function, respectively. The frequencies of this function are $\frac{w}{2\pi}$ and $\frac{w_o}{2\pi}$. The periods of the functions, which are equal to $\frac{1}{Frequency}$, just become $\frac{2\pi}{w}$ and $\frac{2\pi}{w_o}$.

$F_0 Cos(wt)$ is a general equation for a periodic external force [**2**]. By assuming the object is at its equilibrium point, we can set

$$c_1 = \frac{-F_o}{m(w_o{}^2 - w^2)} \quad \text{and} \quad c_2 = 0$$

Using these c_1 and c_2 values, the beats equation takes the form:

$$u = \frac{F_o}{m({w_o}^2 - w^2)} \cdot (Cos(wt) - Cos(w_o t))$$

(8)

These two functions within the beats equation have the same amplitude, just different periods. Referring again to [**2**], we see that the trig identity,

$Cos(A \pm B) = CosA \cdot CosB \mp SinA \cdot SinB$ with $A = \frac{(w_o + w)t}{2}$, and $B = \frac{(w_o - w)t}{2}$, can be used to rewrite equation **8** as a product of Sines:

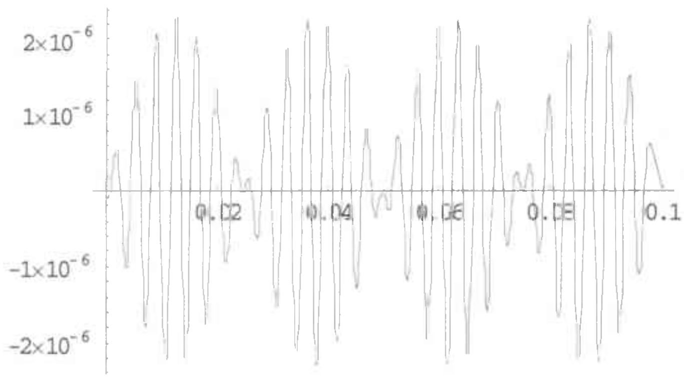
$$u = \left[\frac{2F_o}{m({w_o}^2 - w^2)} \cdot Sin \frac{(w_o - w)t}{2} \right] \cdot Sin \frac{(w_o + w)t}{2}$$

(9)

The authors explain that if $|w_o - w|$ is small, then $(w_o + w)$ will be greater than $|w_o - w|$, and thus we get $Sin \frac{(w_o + w)t}{2}$, which is a function having a high frequency and dynamic amplitude. As one can see, this function has a circular frequency $\frac{(w_o + w)}{2}$. The remaining portion of the beats equation, $\frac{2F_o}{m({w_o}^2 - w^2)} \cdot Sin \frac{(w_o - w)t}{2}$, constitutes the pattern of beats that has a relatively long wavelength and "slowly varying sinusoidal amplitude." This periodic variation of amplitude is called a beat. [**2**]

To illustrate the concept of beats, a graph of the sound made by two tuning forks is shown below in Figure 4:

Figure 4



Looking at the graph, students will observe that the amplitude varies in a periodic fashion. Again, this pattern results from the product of sines shown in the beats equation. If $|w_o - w|$ is small, beats can be observed in music when musicians are tuning their instruments. For instance, our *Overview of Beats and Resonance* explains that " if a musician is tuning his or her flute to a note on the piano and the frequency of the musician's note is slightly different than the frequency of the note produced by the piano, beats can be heard. This signifies to the musician that the instrument is not fully in tune. By adjusting his or her embouchure and/or tuning bar, the musician changes the frequency of the note. The musician can tell the instrument is becoming more in tune as the beats occur farther and farther apart. When the flute reaches proper intonation, beats will no longer be noticeable. Instead, a beautiful resonance will exist, and the musician can begin playing with the band." (See CD Appendix)

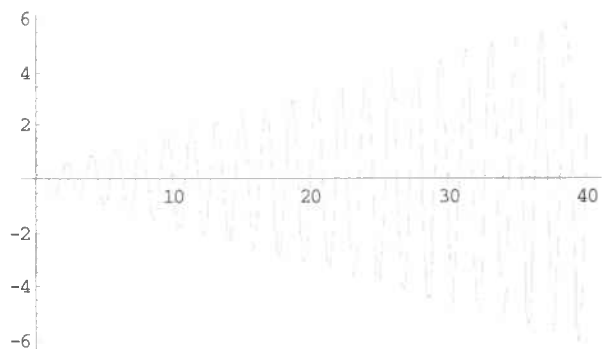
After showing an example of beats in the lab, we proceed to explain the concept of resonance. As one can tell from the overview excerpt above, resonance is the mathematical phenomena created when two forces---the frequency of the forcing function and the natural frequency of the object being studied--- are set equal to each other. Thus let $w = w_o$. The difference between beats and resonance lies here; beats is created when these two forces are very close to each other, whereas resonance is created when they are equal to each other. According to [**2**], the general equation for resonance is:

$$y = c_1 Cos(w_0 t) + c_2 Sin(w_0 t) + \frac{F_0}{2mw_0} t \cdot Sin(w_0 t)$$

(10)

Because of the *(t)Sin(w_0t)* term, as t→ ∞ the amplitude of the wave will increase without bound, regardless of what values we assign to c_1 and c_2. Figure 5 shows an example of resonance.

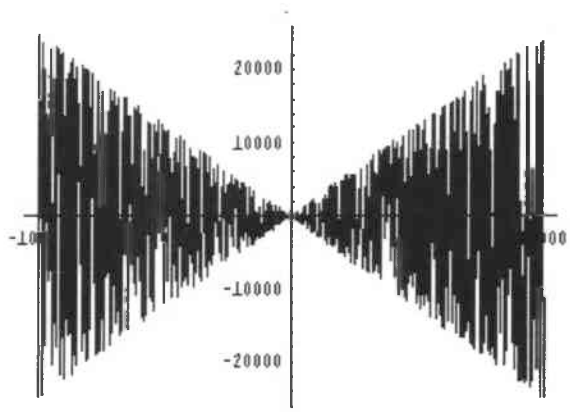
Figure 5



For our example of resonance in the lab procedure, we set our F_0, m, and both c values equal to 1; the graph in figure 6 illustrates this example.

$$y = Cos(5x) + Sin(5x) + \frac{1}{2 \cdot 5} xSin(5x)$$

Figure 6



We encourage students to look at the graphs of beats and resonance to see how they differ. Resonance is an interesting phenomenon; it shows up in many areas in addition to music and can create some serious problems in the real world. For example, when designing structures, like buildings and bridges, one needs to take resonance into consideration. If the wind blows at the same frequency as a high rise building, the structure has the potential to be damaged by excessive vibrations if the builders have not compensated for this. Buildings such as the Sears Tower in Chicago are built with a counter measure, such as weights in the walls of the building that shift, to help prevent such an occurrence. In addition, bridges are also susceptible to the wind frequencies and have been brought down by the wind. Soldiers who march in step with each other will break their stride when marching over a bridge so that they do not cause the bridge to collapse. As one can see, resonance can have serious consequences if it is not taken into account.

What do you think can happen in music due to resonance? Have you ever heard of people being able to break glass by singing? This is one way resonance can cause problems in music, so put away your good china when the kids come over to practice.

At the bottom of the *Beats and Resonance* section, viewers have three choices, they can download an executable *Mathematica* code to their computer, view the code, or return to the *Beats and Resonance Overview* page.

If they choose to view the *Mathematica* code, they will be directed to a page entitled *Beats and Resonance Sound Lab, Viewable Mathematica Code*. In this section, we explain the phenomena of beats and resonance and then review the general equations. Again we present our original beats and resonance examples from the first beats page and have graphed them using *Mathematica* (See CD Appendix). The students are then encouraged to change the values of the mass and forcing function, as well as the c_1, c_2, w_0, and w values, and see how the changes will alter the different graphs. From this page the viewer returns to the *Beats and Resonance Enrichment Lab*.

Beats and Resonance Sound Labs

Our web site includes two additional activities that focus on beats and resonance. They are the *TI-82* and the *TI-92 Sound Lab* exercises. The two labs are almost identical, with the exception of the calculator model required. Teachers may select one lab over the other, depending upon the technology available in their particular schools' classrooms. We chose the TI-82 and TI-92 because these are the two that we had access to and with which we were most familiar, but it should be noted that this program will also work on the TI-83. We did not find any problems when linking the two through the TI link web page on the computer, so any of these calculators will work. The main difference between these calculators is the ability to store and save graphs. As mentioned earlier, the TI-82 will not save the information collected with the graph and data, whereas the TI-92 will. If one uses the TI-82, it is important to transfer data to another source before creating another graph.

We have included both the viewable *Mathematica* code for the sound lab, as well as the capability for downloading the programs from our page. In addition to the lab itself, we also included a sample write-up for the *TI-92 Sound L*ab and possible discoveries students might make when participating in this lab. The write-up is just a sample of the graphs, calculations and possible conclusions one might draw from the lab itself.

When using these labs in a classroom, teachers may be curious to find how many students are interested in music. How many of the students take band or choir? If a teacher can relate math to other fields the students find interesting, it will help motivate their learning. A lot of students and even teachers have math anxiety, but we feel that if people can take math and apply it to their everyday lives and to a subject which they enjoy, it will benefit all who are involved. Making learning fun--- that is what we hoped to achieve when creating this page.

Chapter 4: Links

After exploring through the *Beats and Resonance* section, the viewer may be craving a taste of other math and music topics. To abate the craving, we have provided a hot link to a section filled entirely with a list of links to other sites, which relate math and music in various ways. This section of Internet links represents the final portion of our web page. When we started our web page project, the main goals were to have fun, to make learning interesting for our audience, and to design something that teachers could use in their classrooms as a resource. By including this final page to our site, we feel as if we have done just this.

When surfing the net one can spend many countless hours looking and coming up short. Searching on the Internet was nicknamed "surfing," for a valid reason. One may spend hours searching from site to site, lost in a seemingly infinite ocean of information. Unfortunately, oftentimes very little of the information found is relevant to the topic one was researching. For teachers this can be a quick turn-off to using outside sources in their classrooms. Many educators have little outside time--- who wants to spend that little time searching the net and getting nothing? We felt that linking our page to other related sites would not only increase the usefulness of our page but also help to expand learning on subjects like math and music. We have saved teachers hours of time and made the information readily accessible to them.

Here is the list of links included in this section of the web site:

Math & Music
Math and Music...go hand in hand
Mathematics and Music
Background and History
Beats and Resonance Links
The Mechanical Universe
~ Resonance
~Waves
Exploratorium - Waves & Resonance
Connection to physics
Harmonies and Frequencies Links
The Mechanical Universe
~ Harmonic Motion
What does "well-tempered" mean?
Sound Project
Wavelets vs. Fourier Tansforms
Fourier Analysis
Fourier Series
Jean Baptiste Joseph Fourier (1768-1830)
Other Fun Fact Links
Music and Probability
Sound
Musical Tones

As one can see, we have included links that are relevant to each main section of our web page, as well as links to other trivia-based and fun math web pages. As many people are aware, the web is a fast paced and ever-changing world of information. Web sites come and go frequently. Due to this, we have found that many of the sites to which we originally linked are no longer working, thus we have removed outdated sites from our link page. We will continue to update our links as the web changes; this may entail deleting links that are no longer in service, as well as adding new links that incorporate our subject.

Summary Observations

For educators who are either unsure about allowing students free reign on the computers *or* for those who teach at schools that do not have Internet access, we offer a possible solution---or at least something to think about.

As far as the free reign of students is concerned, this will take some monitoring on teacher's part. However, if the students are aware of the expectations, monitoring should not be too difficult. Many parents and teachers express concern over students finding unreliable or inappropriate sites on the Internet. Although this is a valid concern, with the proper guidance, students can enhance their education and even be taught how to search for reliable sources. Still, the worry in the community over faulty and inappropriate information may be too great to let students search the Web on their own. Therefore, teachers can limit the sites searched by *saving* the subject-related web sites on disk and using the stored information in a classroom without even going on-line.

Using stored information will also act as a solution for teachers who wish to use the Internet but have no Internet connection in the classroom. If computers are completely lacking in the classroom, one can print out our labs from another computer and take them back to the classroom. Students can complete all of the labs using their calculator and some extra equipment. Although computers clearly enhance the lab procedures, students can visualize a majority of the concepts through the use of the CBL's and the calculators. In addition, the teacher's print-out of the viewable *Mathematica* code would enable students to see what graphs generated by the computer would look like.

There are many ways to integrate the World Wide Web into an educational program. We feel that integrating technology into the classroom will help teachers illustrate concepts to students. The ideas we developed are only a few of the possible ways to accomplish this. As prospective educators, we feel the Internet will be an integral part of education in the future and will be used even more effectively and more often than it is now. The Internet is an excellent way to motivate students to learn and extend the concepts taught in the classroom into other disciplines. Through this experience students not only discover new aspects of mathematics, but are also exposed to operating the computer and using the Internet for multiple purposes. Through these lessons, students advance their skills, confidence, and knowledge of the computer and the Internet.

There is no limit to the applications of the Internet. The lessons and information available in all disciplines is unlimited and many interdisciplinary approaches to each lesson exist. We challenge teachers, students, and other school personnel to explore the integration of the Internet into their own school classrooms.

Another idea for integrating the Internet into the classroom is to introduce the students to the process of developing their own web pages. With very basic knowledge, one can begin to develop a very imaginative and functional web page. The students may take this idea and decide to post a project that they have completed or share data with other students.

Classrooms from different schools are now beginning to work together on projects using the Internet. Resources are available to help educators learn how to use the Web for these types of projects. For instance, *The Educator's Guide to the Internet*, compiled by the Virginia Space Grant Consortium, offers a list of reliable web sites and explains some of the basics of the Web [7]. Also, [7] includes activities designed for students from kindergarten through grade 12. One popular idea in these activities is to have students in one school communicate with students in another school via e-mail using resources such as *KidsNet* [7]. A variety of activity topics are available; one example in [7] suggests having second-graders in the United States study weather patterns by communicating with students in different geographical areas, such as South America.

The opportunities and ideas are unlimited. We challenge you to get involved with integrating the web into your classroom. Start by simply gathering facts and data from various Internet sites. Then begin finding and using ideas or lessons already on the Internet. Next, try to develop your own lesson plan integrating the Internet and explore the idea of educating yourself on web page development in order to share your own lesson plans with others. Finally, challenge your own students to explore the idea of developing their own web pages to gather and present information.

Conclusion

While developing this project, we shared a number of learning experiences. For instance, we learned that patience and creativity can solve many problems associated with using technology. Although technology has wondrous advantages, it has its glitches, too. Whether struggling to put *Mathematica* code into html format or trying to fix the paper jam in the printer, we discovered that solutions were not far out of reach if we implemented a bit of renaissance spirit.

In addition to learning how to troubleshoot and overcome technical difficulties, we found working in a group to be challenging at times, simply because of scheduling conflicts and the need for long-distance communication. For instance, Kayme had to complete a semester-long student-teaching internship, while still finding time to trek from Great Falls to Helena in order to meet the thesis group. Kimberly balanced her student-teaching internship with a part-time job, as Marisa juggled classes at Carroll College and two part-time jobs. Consequently, finding a suitable meeting time for all members presented a challenge. Fortunately, technological advances such as electronic mail enabled us to communicate about the thesis easily, and the weekends provided a common meeting time for group work. In short, creating the web site has prepared us for group efforts such as team-teaching and committee projects, which we will participate in as teachers.

Another benefit of this experience is the skills we gained in developing web pages. To our surprise, we discovered that a web page can now be designed with relative ease, using software tools such as *Netscape Editor*. Also, we learned how to search more efficiently for information on the World Wide Web as we found sites relating to mathematics and music.

In the process of our search, we began to realize the massive amounts of *reliable* information available on the Web and learned what types of sites are trustworthy. Often teachers and parents express their concern over children accessing inappropriate sites or sites full of faulty information. Our experience with this web project will enable us to put parents' and teachers' concerns to rest by teaching students how to use the Web effectively, appropriately, and efficiently.

Overall, participating in this project has been a positive experience. We have tried to convey some of the many ways to integrate technology into the classroom. In addition, we hope to have sparked an interest in teachers, so they will explore the technology available in their schools and use it to enhance their students' quality of education.

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