Exploring Interactive Math Activities for the Classroom

Reagan B. Ryan

Student Undergraduate Research Paper

Department of Mathematics Carroll College

Senior Thesis

Advisor: Jodi Fasteen, Ph.D

December 8, 2024

Abstract

As the ideas of how best to educate students have evolved, the ideas behind creating engaging math tasks have also changed. Luckily, there has been a significant amount of research on how to develop good math tasks. This paper describes several characteristics of engaging math tasks and justifies each characteristic with research. Then, three example tasks are provided with these characteristics in mind. One task is created for Geometry, one task is created for Algebra 1, and one task is created for Algebra 2.

Table of Contents

Introduction

When parents send their child to school, they hope their child gains skills that will last a lifetime. The long-term goals of education are to prepare individuals to be problem solvers and active contributors to society. Through a combination of teaching new concepts, reinforcing previous knowledge, and engaging students in thought-provoking activities, teachers attempt to accomplish these goals. Math activities are one important tool in increasing student achievement and understanding.

Designing effective math tasks for students of any grade level can be daunting. Educators may wonder if the tasks they use are the most effective in either teaching a new concept or reinforcing an older idea. While every classroom is different, countless studies have been conducted on designing math tasks that support student learning. This paper will begin by providing characteristics of good math tasks, and research that supports each characteristic. Then, a math task that applies these characteristics will be provided for Algebra 1, Geometry, and Algebra 2 courses.

Characteristics of a Good Activity

Throughout the coursework of an education degree at Carroll College, many ideas are taught to pre-educators about how to create successful lessons. Furthermore, wide-ranging research has been conducted on designing mathematical tasks. A good math activity engages students in the content and leads to student learning and retention of concepts. This section synthesizes the most common characteristics of high-quality math activities supported by research.

The first characteristic of a good math activity is that it is meaningful to students. Highly famous and respected Carroll College Professor, Dr. Cline often reminded pre-service teachers that “half of your job [as a teacher] is to be a salesman, you have to convince your students that what they are learning is useful” (Kelly Cline, personal communication, Spring 2024). When using tasks that students do not find meaningful, they are less likely to see how the math topic helps them. They may learn the skill but are less likely to know how to apply the skill to real-world scenarios. The book Guidelines for Assessment and Instruction in Mathematical Modeling Education suggested using “common, everyday questions to motivate the use of mathematics” (Teague et al., 2016, p. 64). Exploring these questions draws parallels to student interests and the math behind them.

The second characteristic of a good math activity is that it should allow students to make and learn from meaningful mistakes. Making mistakes in math is inevitable, but mistakes are only meaningful if a student can learn from them. Chan and Leung found that teachers can construct tasks that have a higher possibility of making [meaningful] mistakes (2013, p. 64). These mistakes can “initiate mathematics discussion which leads to the construction of mathematical knowledge” (p. 43). A math task that exposes a common misconception can force students to ponder concepts in a somewhat controlled environment. If students do not make mistakes, there is no loss, as the students demonstrate how to complete the task. However, if they do make the mistake, then the teacher can explain why their approach to the problem was wrong, and steer them in the right direction. Therefore, if a task creates opportunities for students to make mistakes, the instructor can facilitate a conversation that could increase student learning.

The third characteristic of a good math activity is that its assessment is focused more on the process than the answer. Traditionally, the most important aspect of grading an activity is the result. A student who arrives at the correct answer should surely deserve a better grade than a student who arrives at an incorrect answer. However, consider a student who displayed excellent mathematical reasoning throughout solving a problem, but wrote that two times three equals five at one point in their work. This mistake causes them to get the wrong answer. Should they get docked heavily because they reached an incorrect answer even though their process was correct? Furthermore, consider the student who did not display a good process but still had the correct answer written on their paper. Did they demonstrate more knowledge about the topic compared to the previous student?

In mathematics, assessing the process is important in determining how well a student understands a concept. The process is how students communicate their math, allowing them to justify their final answer. Furthermore, explaining a process helps them remember it for future use. If the process is thorough, it can be replicated by students and their peers. While getting the right answer is important, the process allows students to communicate their learning and replicate it in the future.

These scenarios often play out in math classrooms today, which is why many researchers suggest emphasizing the process more than the result. Dindyal (2013) suggested when assessing problem-solving tasks, “students should see that they earn enough credit for specific steps and/or intermediate strategies that they use” (p. 316). This can be achieved by using a rubric or a checklist for completing different aspects of a problem. Furthermore, Bliss and Libertini (2016) suggested that in modeling tasks, “assessment should focus on the process … which will help students' ability to model, which will, in time, translate into a better product" (p. 21). As students learn how to reflect on their model-building process, their results should improve.

The final characteristic of a good math task is encouraging students to collaborate. This characteristic is not exclusive, meaning there can be good individual tasks for students. Collaboration as a characteristic does not mean that students always have to work in groups; rather, students are put in a position where they can talk through problems with their peers. This discussion allows students to see and learn from the perspectives of their peers. Research suggests that collaboration helps students succeed. For instance, in modeling tasks, Bliss and Libertini (2016) claimed that student collaboration "helps [students] know that they have support as [they] seek answers" (p. 21). The effect of this support was seen in a study by Abd Algani. This study split 280 8th-grade students into two groups. One group was taught through the school's traditional methods, and the other was taught through collaborative learning. They found a statistically significant difference between the two groups, allowing them to conclude that "using the collaborative learning approach has a major [positive] effect on students' math performance" (2021, p. 101).

Example Tasks

Now that a few characteristics of good math tasks have been defined, they can be applied to three math tasks. Tasks for Geometry, Algebra, and Pre-Algebra are described in the following sections.

The City Game

Research:

The inspiration for the first activity came from an observation of a Geometry class in the Fall of 2023. The activity was created by Ross Gustafson in 2016 and has been taught at Capital High in Helena Montana ever since. The purpose of the City Game is to provide engaging practice of the different angle relationships such as corresponding angles, vertical angles, alternate interior angles, alternate exterior angles, same side interior angles, and same side exterior angles. This activity only fulfills teaching the angle relationships required to prove theorems in the standard listed.

This activity utilizes the characteristics of a good activity described above. First, it presents a meaningful problem, as students need to be comfortable reading maps in their everyday lives. Second, it allows the opportunity for meaningful mistakes by having two sets of parallel lines where each line can act as a transversal depending on which area the students are looking for. This can lead to the common mistake of thinking that there is one additional option to place locations in their city when there is not. This mistake is described in more detail in the anticipated mistakes section. Third, this activity can be assessed on its process. The teacher can observe students' thoughts as they work together to place locations in each city. Finally, the City Game allows students to collaborate with their peers by working in groups to complete their city.

Standards: Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent, and corresponding angles are congruent (G-CO.9).

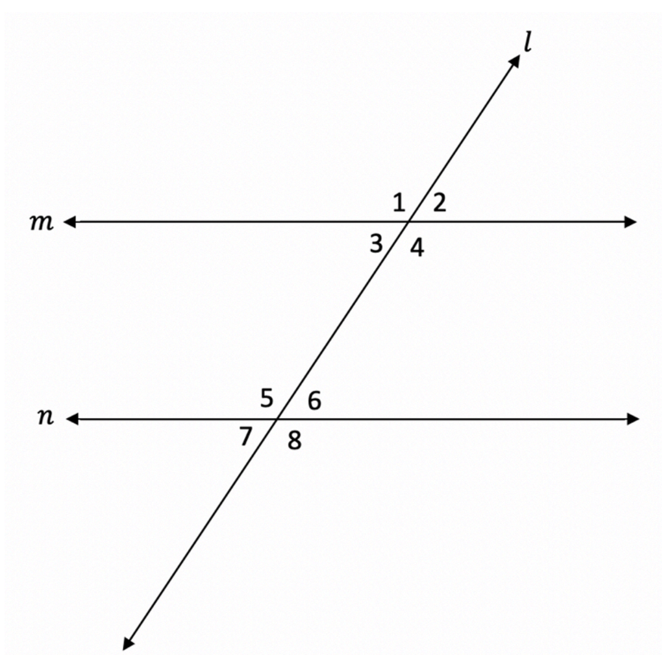
Timeline:

- Introducing Background Knowledge: 5-10 minutes
- The Game: 10-15 minutes per city.

Materials:

- Posters for various cities (see Appendix A)
- Flashcards of Locations for each city (see Appendix A)
- Key for Each city (see Appendix A)

Introducing Background Knowledge: Review the following angle relationships using the following figure: vertical angles, corresponding angles, alternate interior angles, alternate exterior angles, same side interior angles, and same side exterior angles. Ask students to identify a pair of each angle relationship in the picture. An example is as follows: Name 2 sets of vertical angles. Answers include 1 and 4, 2 and 3, 5 and 8, and 7 and 6.



Notice!

Notice lines *m* and *n* are parallel to each other and cut but the transversal line *l*, forming angles 1 through 8.

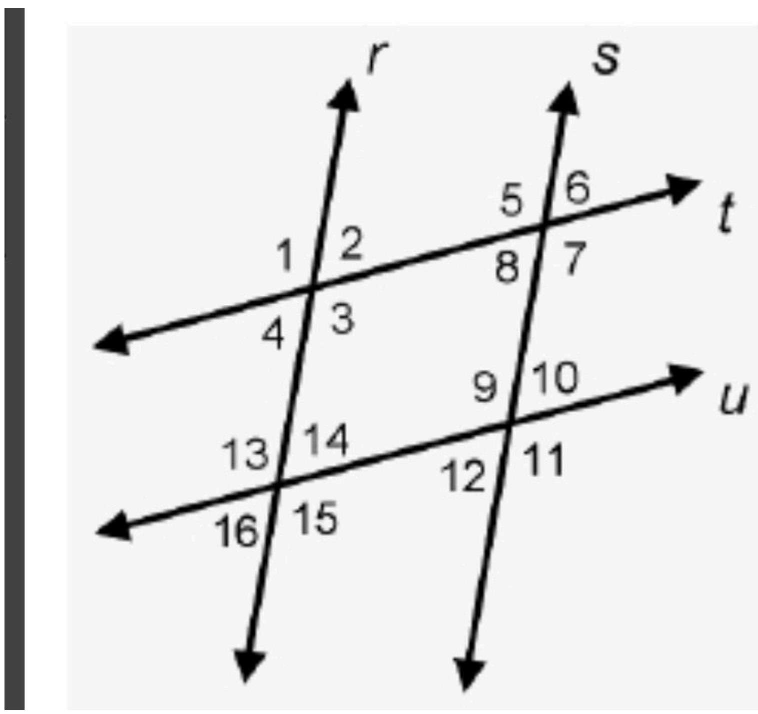
Introducing the Game: The teacher can divide students into groups of four to six using their preferred grouping method. Groups of four are ideal as each student can have four locations, but this may not be practical in larger classrooms. Then, the teacher may give each group a posterboard of a city. The teacher can explain to groups that they will be given a set of sixteen flashcards corresponding to the city's locations. They will need to divide the flashcards evenly among groupmates. So, if there are groups of four, each member has four flashcards. If there are groups of five, each member will have three flashcards and one member will have four flashcards. Each group member can only touch the flashcards they are given, encouraging every person to participate. The front of the flashcard lists the location, and the back lists the directions that tell students where to put the location in the city. One building will be a freebie, as the directions will tell students exactly where to place that city. From there, students will need to use their knowledge of angle relationships to place the rest of the buildings.

Once students have started working, the teacher may monitor each group's progress. Groups may struggle to find the next location that can be placed. The teacher may let them struggle for a few minutes but eventually provide guidance in the form of "I see you have placed _______ location. Which of the flashcards mentions the location you just placed?" The teacher should also monitor if students are on the right track early and often. If students make one mistake, many locations in their city will be misplaced, and they will have to backtrack until they find their mistake. Therefore, if the teacher observes a location in the wrong spot, they should help correct the mistake.

Anticipated Mistakes:

1. The anticipatory set has one set of parallel lines and one transversal. However, each city has two sets of parallel lines, creating multiple transversals, and requiring students to identify the transversal and which set of parallel lines to look

at when placing a location. When consulting the figure below, students will characterize additional angles as an angle relationship when the relationship does not apply. For instance, when identifying a corresponding angle with angle number 1, they may correctly cite angles 13 and 5. However, they might cite angle 9 as corresponding to angle 1, which is incorrect. This mistake will cause students to get stuck and not know where to place some buildings, as they may think there are three options when there are only two possible locations from the angle relationship.



2. Another mistake can be simply not reading the directions carefully. Students may hear the angle relationship but not consider the street location, causing them to misplace a building.

Teacher Modifications:

1. The flashcards with the cities on them are intentionally not numbered to add a problem-solving and collaborative aspect to the game. If a teacher were short on time and wanted to reinforce the concept over the problem-solving aspect, they could number the flashcards.
2. If the teacher wanted students to go through the struggle even more, they could not help students but allow each group to buy one or two hints if they get stuck. This idea will enable students to be strategic when they ask for help and try to fix mistakes on their own.
3. This activity could also be a race to see who finished first, with the winner receiving extra credit, a homework pass, or some other reward.

Buying a Car Project

Research:

Owning a vehicle is an essential aspect of adulthood, as the US Census Bureau found that around 92% of American households have at least one vehicle (2023). Sullivan et al. (2020) from the Federal Trade Commission conducted a study using in-person interviews of forty recent car buyers. Since their sample size was small, the purpose of this research was to provide insight into the knowledge of car buyers. They found that buyers focused on finding a specific monthly payment, rather than “considering how the overall cost of the financing depends on other factors such as the interest rate, down payment, maturity, and amount financed” (p. 17). In other words, buyers are often focused on only one aspect of the financial impact of the car purchase, rather than the entire picture. This demonstrates the importance of teaching how the total cost of a car loan changes as financing terms change.

Research suggests teaching this real-world scenario through a mathematical modeling perspective benefits students. For instance, Wethal (2011) conducted an eight-week study of 77 juniors and seniors enrolled in Algebra 2. Through a pre and post-analysis, she found that using mathematical modeling tasks allowed “students [to] recognize a positive impact of the mathematical modeling process and how it relates to their learning” and caused “students [to be] more willing and able to try new problems” (p. 52). She also noted that using mathematical modeling had an “impact on student attitudes and learning” (p. 53).

Santos et al. (2015) compared the success of using traditional methods versus mathematical modeling tasks in ninth grade. They had the same teacher teach the same material to both the control and experimental groups. They found that using mathematical modeling was more effective in “teaching problem-solving topics in mathematics” (p. 112). They also found that modeling tasks were “more effective in reducing the … students’ math anxiety level” (p. 112-113).

From these pieces of research, it can be concluded that mathematical modeling is an appropriate method to teach students about the math behind buying a car. An example car buying project that considers the research above is shown below. This project aims to expose students to the mathematics behind purchasing a vehicle. Students will model how long it takes to pay off a car loan over the life of the loan. Through this process, students will learn how to use spreadsheets as a modeling tool. This activity will focus on Google Sheets as it is free and easily accessible.

This activity fulfills the four characteristics of good math activities described at the beginning of this paper. Buying a car is a meaningful problem for students in Algebra

2, as many of them are driving or are taking driver's education classes. This activity also presents opportunities for meaningful mistakes through an inquiry approach to teaching the use of spreadsheets and creating a model for car payments. This activity can be assessed on the process of creating the model, as the teacher can see the students' model. Students will communicate their thinking through short answer responses, allowing the process to be properly assessed. Finally, the car project allows collaboration in the first section, where everyone has the same problem and students are encouraged to ask questions of their peers.

Standards:

- Create equations and inequalities in one variable and use them to solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. (A-CED.1)
- Write a function that describes a relationship between two quantities. ★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. (F-BF.1)
- Identify problems suited for technology-assisted methods for data analysis, abstract models, and algorithmic thinking. (TI.CT.9-12.1)
- break down problems into component parts, extract key information and develop descriptive models to understand complex systems or facilitate problem-solving. (TI.CT.9-12.3)
- HS Conceptual Category: Modeling

Timeline:

This activity will take approximately three to four days to complete, and the allocation of time is shown below.

- Introducing Background Knowledge: 5-10 minutes for discussion of loan terms
- Introducing Background Knowledge: 20-30 minutes of introduction to using Google Sheets.
- Completing the Student Activity: 2-3 fifty-minute class periods.

Materials:

- Student Handout (see Appendix B)
- Teacher Example Handout (see Appendix B
- Teacher Example Spreadsheet (see Appendix B): see excel sheet labeled Car_Project_Teacher
- Rubric (see Appendix B)

Introducing Background Knowledge:

The anticipatory set should depend on students' prior knowledge of two items: loans and Google Sheets. If students have never seen information about loans, then a lesson that introduces terms like sticker price, downpayment, principle, and interest rates (yearly versus monthly) may be needed. The teacher can begin by writing these terms on the board. The teacher can then place students in groups of 3-4 and ask them to discuss their prior knowledge of these words with each other for a few minutes. While students are discussing, the teacher may listen in on their conversations to identify correct ideas and common misconceptions about those terms. Then the teacher can bring the class together to clearly define these terms. The following definitions can be used:

- Sticker price: the amount the dealer charges for a vehicle

- Downpayment: how much the buyer pays on the day they purchase the vehicle
- Principle: The amount the buyer is borrowing from the bank or lender.
- Interest rate: a percentage of the loan charged to the borrower as interest, which can be expressed yearly or monthly.

The other piece of background knowledge that may be required is an overview of how to use Google Sheets as a modeling tool. If students have not used Google Sheets before, an overview is recommended and will take longer than the typical anticipatory set. The Teacher Example Google Sheet has a sheet labeled “Anticipatory Set.” The teacher may walk the students through how to create the items in that sheet, and why it is useful. The teacher may consider following the steps below and having students model these steps with the teacher. If the teacher is using a different spreadsheet tool, they may modify the ideas below for their spreadsheet tool.

1. The teacher can begin by giving a tutorial on how to get to Google Sheets. The teacher can search “Google Sheets” in their web browser. The teacher can sign in and create a blank spreadsheet.
2. The teacher can explain the orientation of Google Sheets. Specifically, they may mention that the columns are labeled as letters “A, B, C,” and the rows are labeled by numbers “1, 2, 3.”
3. The teacher may explain that Google Sheets can be used as a calculator to complete math problems by using the equal sign followed by the expression students typically would type into a calculator. The teacher can review the examples in column A of the Anticipatory Set Sheet.

4. Next, the teacher can explain how to reference cells and do calculations with those numbers, as seen in columns B and C. The teacher may change the numbers of the variables *a* and *b* to demonstrate that Google Sheets can do the calculations for us when we change the values of a variable.
5. Now, the teacher may use Google Sheets to create a table of values representing the function $y = 2x + 1$, as shown in columns E and F. The teacher may explain that if a formula is applied once referencing an *x* value, then dragging the formula down results in the formula being applied to the corresponding input value.
6. Then, the teacher may use Google Sheets again but reference the slope and y-intercept as variables as shown in columns G-K. The teacher may demonstrate why the error occurs in column J by double-clicking on cell J3 showing students which cells are being referenced, and asking students which cells are not referenced correctly. The teacher can explain that adding dollar signs will lock the referenced cells in place when the formula is dragged, representing this in recreating column K.
7. Finally, the teacher may change the slope and y-intercept, once again demonstrating to students that Google Sheets will do the calculations for them.

If students have used Google Sheets before, then a shortened version of this anticipatory set is recommended. This could involve asking students to use Google Sheets to create a table of values for a linear function. Then, the teacher could model steps 6 and 7 of the previous anticipatory set to ensure students see a demonstration of referencing cells and using dollar signs to lock in cells as needed.

Introducing the Project:

To introduce this project, the teacher may hand out students copies of the project to each student or post it online on the classroom interface. They then may explain to students that this project's purpose is to explore the math behind car loans so when students buy a car, they are well-informed about how car loans work. The teacher can explain that the project has two parts. In the first part, they may collaborate with a partner to solve a story problem related to John's purchasing story. In the second part, they will independently research two of their dream cars, and determine which car they should buy.

After assigning the project, the teacher should frequently check in with students. If the teacher notices a mistake, the teacher could call students' attention to the mistake, and guide them to the correct answer. Part 1 Question 5 has students check their work with the teacher. This serves as a formative assessment, where the teacher can check to see if the students are on the right track, and can correct mistakes or misconceptions as needed.

Anticipated Mistakes/Questions/Concerns:

1. This project relies heavily on using Google Sheets, likely a new technology for many students. Students may get easily frustrated when using new technology. When this frustration occurs, the teacher should intervene and help the student with their questions. Furthermore, the teacher needs to remind students that they are not expected to know how to use Google Sheets. Instead, they will learn how to use Google Sheets in this project. When creating their models in Google Sheets, a few common technical mistakes are listed below.

a. A frequent mistake students will make is in using dollar signs when locking in cells. They will either lock in a cell that should not be locked or they will not lock in a cell that needs to be locked. This can be identified by observing that the calculations look correct in the first row but go crazy or zero out in the following rows.
b. Another mistake students may make is accidentally referencing the wrong cell in their sheet.

2. When students start part 2 of the project, they may ask questions about what down payment or monthly payment to choose. This part of the project is open-ended so students can be creative and feel like they are making genuine choices. Therefore, the teacher could encourage students to research or be creative in selecting these items. If students are still stuck, the teacher can guide them by asking questions such as “What is the average car payment per month?”

Teacher Modifications:

1. If students do not have computer access, then they can be partnered and work on one sheet. The following direction can be added to part 2 so both students participate in building the model. One student will type while the other will dictate the first dream vehicle. Then the students will switch roles for dream vehicle number 2. They will have to decide which vehicle they should buy and under what conditions.
2. If time is an issue, then questions 13 and 14 could be removed and students could only analyze one dream vehicle. If time is still an issue, part 1 could be assigned and part 2 could be revisited later. Cutting all of part 2 is not recommended

because that is when students get to be creative and select a vehicle that interests them.

3. Another cut that could save time if needed is cutting the gas aspect of the project. This is questions 7-12 in part 2. This would allow students to explore their dream car and save time.
4. The teacher could choose to add two presentation days if time allowed. Students could prepare a 3-4 minute presentation on the two cars they examined, which one they chose to buy, and why they made that decision. The teacher should consider timing students' presentations and cutting them off after 4 minutes to finish within two days.

Discovering the Concept of Slope

Research:

Slope is a crucial concept for students to understand as they move forward in future math classes and understand trends in the world around them. Therefore, it is crucial to use research-based practices when teaching this concept. Nusantara and Putri (2018) analyzed the effect of utilizing triangles to develop an understanding of rise and run, and how to put those topics together. They concluded that looking at triangles helped students develop an increased understanding of slope as a rate of change. Furthermore, Smith et al. (2013) researched effective ways to teach slope. They had students measure the steps on staircases to determine how steep they were. They also had students make observations about which dimensions of a staircase could be changed to make the staircase steeper. They found that having students use staircases helped students “develop an understanding of slope and rate of change as a ratio” (p. 376). This helped students “transition to the coordinate plane to develop the formula to calculate slope” (p. 376). This research supports utilizing triangles and staircases as tools to develop the concept of slope.

Another question is which teaching method is best for exploring middle school concepts like slope. Noreen et al. (2019) conducted a study of 120 seventh-grade students randomly divided into two groups. One group was taught with an inquiry and activity-based teaching approach whereas the other group was taught with a traditional approach. Through the use of pre and post-tests, the researchers found a significant difference in the performance of students who received the inquiry-based approach versus those who received traditional teaching methods, allowing the researchers to conclude that “students taught through activity-based teaching [have] strong power of retention” (p. 155). This research supports using an inquiry and activity-based approach in teaching concepts to middle school students.

An example lesson plan for teaching slope with these research methods in mind is shown below. This lesson plan follows the four characteristics of a good math activity described in the first section of the paper. It presents a meaningful problem by asking students which ramp or staircase they would rather walk up. It allows for meaningful mistakes through the inquiry process of developing the process for finding the slope. This lesson also focuses on developing the process for finding slope, so it is assessed on the process over the answer. Finally, this lesson utilizes collaboration throughout the lesson in developing the process of finding the slope of a line. The purpose of this lesson is to provide a foundation for students encountering slopes for the first time.

Standards:

- Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★ (F-IF.6).

Timeline:

- Introducing Background Knowledge: 5-10 minutes
- Lesson: 35-40 minutes

Materials:

- Student worksheet (see Appendix C)
  - The implementation of staircases in this activity was inspired by Math Giraffe (2016).
- Teacher Slideshow (see Appendix C)

Introducing Background Knowledge:

The teacher may write the following fractions on the board and ask students to simplify them: $\frac{-4}{-8}$, $\frac{-6}{18}$, $\frac{6}{10}$, $\frac{0}{12}$.

The teacher can give students a few minutes to simplify these independently. Then, the teacher can review the answers with the entire class. Common errors on this task include struggling with the negatives. The teacher may reinforce that a negative divided by a negative is a positive, but a negative divided by a positive is a negative. Another mistake could be simplifying $\frac{-6}{18}$ to be $\frac{-3}{9}$ and stopping there. This is due to identifying the greatest common factor as 2 instead of 6.

Instruction:

The teacher may begin by giving students slide two of the slideshow. The teacher may split students into groups of two or three using any grouping method. The teacher can have students work in groups to decide which ramps and staircases are the easiest or hardest to climb. The teacher may let them work in their groups for about 5 minutes, and eavesdrop on their conversations. Then, the teacher can bring the whole class together and ask a few groups who have differing opinions to share their reasoning with the class.

After this discussion, the teacher can hand out the slope student copy worksheet and ask students to turn to the page with ramps and staircases (slide 3 of the slideshow). The teacher may have students work in groups to attempt to assign a number to the steepness of each staircase and ramp. The teacher could sit back and let them work. If students are still struggling, then the teacher may offer encouragement. After 15-20 minutes of exploration, the teacher may point out the strategies students used. The teacher may consider emphasizing strategies that result in the slope being written as a fraction. Hopefully, some students will count how many units up and how many units over to go from one step to another, or from the bottom of a ramp to the top. The result of this operation is two numbers, and the teacher may challenge students on how to report their answers. Should it be units up over units over or vice versa? The teacher can have students discuss this question in groups again. Then, switch to slide 6, and the teacher may have students calculate the four slopes requested. Next, the teacher may discuss the calculations of these slopes together in class and switch to slide 7. The teacher can have students discuss which measure should be used.

Then, the teacher may draw the student's attention to the direction of the lines. The teacher can ask groups if there is a difference between the slopes of figures B and F. The teacher may have students discuss this idea, and may once again bring differing opinions to the class to discuss. The teacher may make the comparison to reading, where students always read left to right, and say that for consistency, we should apply the same logic here. The teacher may also make a comparison to the number line, saying that the numbers increase as we read from left to right. Then, the teacher may ask students how this changes their slopes from before.

Finally, the teacher can conclude the lesson with a check for understanding shown on slide 11 and page 3 of the student handout. Since this lesson is just the beginning of students' practice with slopes, we are not expecting a complete understanding of the concept. It is recommended that students get a day to complete practice problems to work on with teacher supervision, so the teacher can correct common mistakes.

Conclusion

Creating a math task can be challenging for current educators and curriculum designers. However, a good math task can be created if the following characteristics are kept in mind: the task should make problems meaningful to students, allow students to make and learn from meaningful mistakes, be assessed more on the process than the answer, and encourage students to collaborate. Since educators do not have the time to create every task they use in the classroom, they should keep these characteristics in mind when looking online for tasks.

References

Abd Algani, Y. M. (2021). The effect of the collaborative learning technique on students' educational performance in math. Journal for the Mathematics Education and Teaching Practices, 2(2), 81-91.

Bliss, K., & Libertini, J. (2016). What is mathematical modeling? In *GAIMME: Guidelines for Assessment & Instruction in Mathematical Modeling Education* (pp. 7–21). Consortium For Mathematics And Its Applications; Philadelphia, Pa.

Chan, Y.-C., & Leung, A. (2013). Rotational symmetry: Semiotic potential of a transparency toolkit. *Task Design in Mathematics Education. Proceedings of ICMI Study 22*, 37–46. https://hal.science/hal-00834054v2/file/ICMI_STudy_22_proceedings_2013-FINAL_V2.pdf

Dindyal, J., Tay, E., Quek, K., Leong, Y., Toh, T., Toh, .P, & Ho, F. (2013). Designing the practical worksheet for problem-solving tasks. *Task Design in Mathematics Education. Proceedings of ICMI Study 22*, 315-326. https://hal.science/hal-00834054v2/file/ICMI_STudy_22_proceedings_2013-FINAL_V2.pdf

Math Giraffe. (2016). *Guide to teaching slope of a line*. https://www.mathgiraffe.com/blog/complete-free-guide-to-teaching-slope

Noreen, R., Majid, A., & Rana, K. (2019). Activity-Based Teaching versus Traditional Method of Teaching in Mathematics at Elementary Level. *Bulletin of Education and Research*, *41*(2), 145–159.

Nusantara, D., & Putri, R. (2018). Slope of straight line in ladder: A learning trajectory. *Journal of Physics: Conference Series.*

Santos, M. L. K. P., Belecina, R. R., & Diaz, R. V. (2015). Mathematical Modeling: Effects on Problem Solving Performance and Math Anxiety of Students. *International Letters of Social and Humanistic Sciences*, *65*, 103–115. https://doi.org/10.18052/www.scipress.com/ilshs.65.103

Smith, T. M., Seshaiyer, P., Peixoto, N., Suh, J. M., Bagshaw, G., & Collins, L. K. (2013). Exploring slope with stairs & steps. *Mathematics Teaching in the Middle School*, *18*(6), 370–377. https://doi.org/10.5951/mathteacmiddscho.18.6.0370

Sullivan, M., Jones, M., & Reynolds, C. (2020). *The auto buyer study: Lessons from in-depth consumer interviews and related research joint staff report of the bureau of economics and bureau of consumer protection federal trade commission*. Federal Trade Commission. https://www.ftc.gov/system/files/documents/reports/auto-buyer-study-lessons-depth-consumer-interviews-related-research/autobuyerstudyjointreport.pdf

Teague, D., Godbold, L., Malkevich, J., & Kooij, H. (2016). Mathematical Modeling in High School: Grades 9 through 12. In *GAIMME: Guidelines for Assessment & Instruction in Mathematical Modeling Education* (pp. 45–68). Consortium For Mathematics And Its Applications; Philadelphia, Pa.

U.S. Census Bureau, U.S. Department of Commerce. (2023). Physical Housing Characteristics for Occupied Housing Units. *American Community Survey, ACS 1-Year Estimates Subject Tables, Table S2504*.

https://data.census.gov/table/ACSST1Y2023.S2504?q=car ownership percentages.

Wethall, N. (2011). The Impact of Mathematical Modeling on Student Learning and Attitudes. *The Evergreen State College*.

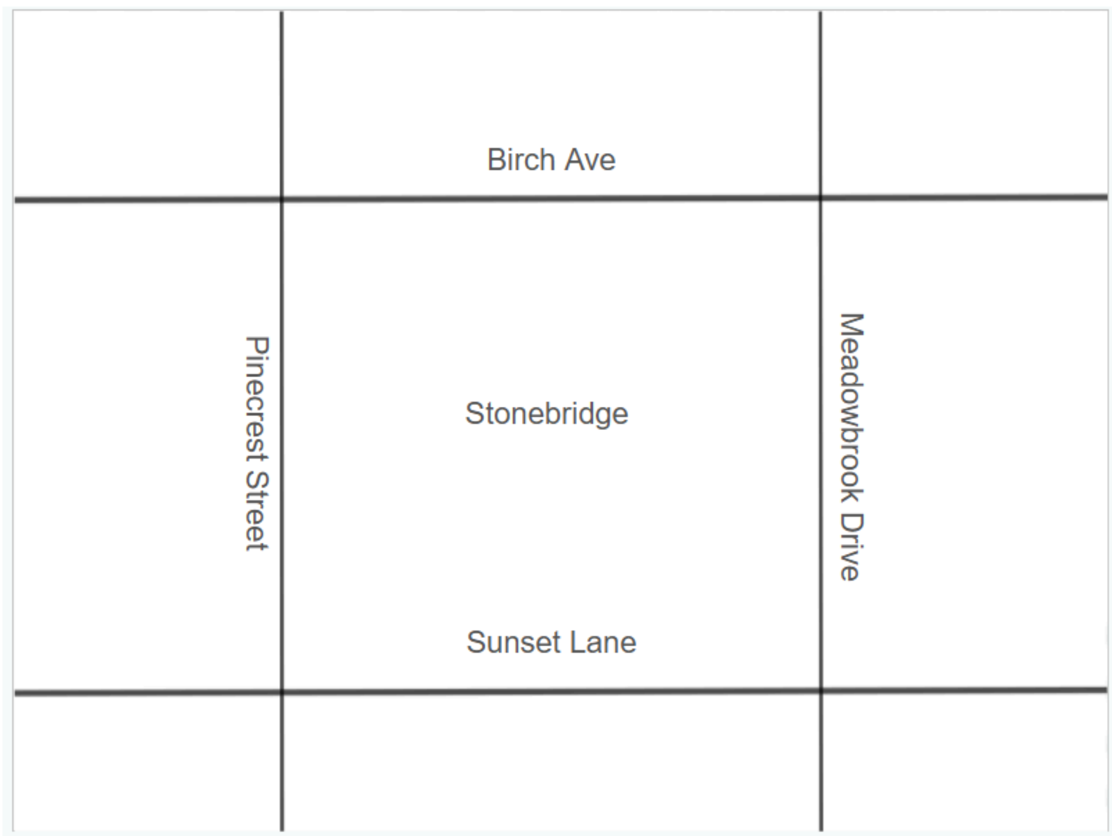
Appendix A: City Game Materials

Posters

This section of **Appendix A** shows the posters that will be given to students in the City Game. **Figure A.1** and **Figure A.2** show screenshots of what the cities look like for reference.

Figure A.1

Stonebridge City Game



Note. This figure shows a screenshot of the poster students receive when playing the city game with the city of Stonebridge.

Figure A.2

Ironwaste City Game

Note. This figure shows a screenshot of the poster students receive when playing the city game with the city of Ironwaste.

Flashcards

This section of **Appendix A** shows the flashcards that will be given to students to fill in the city maps shown above.

Stonebridge

The names of the locations were generated by the following link:

https://chatgpt.com/share/4c2b4ee0-7a66-47f7-85bf-7f8cbdea8156

The Corner Café	Greenfield Park	Sunrise Elementary School
Lakeside Community Center	Riverside Library	Hilltop Fire Station
The Village Market	Horizon Fitness Center	Northside Pharmacy
Central Dog Park	Evergreen Flower Shop	Horizon Medical Clinic
Mountainview Playground	Central Post Office	The Tool Shed
The Oaks Apartment Complex		

The Corner Café is located on the Northeast corner of Meadowbrook Drive and Sunset Lane	Greenfield Park forms a corresponding angle with the Corner Café and is on Pinecrest Street	Sunrise Elementary School forms an Alternate Interior Angle with Greenfield Park and is on Sunset Lane
Lakeside Community Center forms an Alternate exterior angle with Sunrise Elementary School	Riverside Library forms a vertical angle with Lakeside Community Center	Hilltop Fire Station forms a same-side interior angle with Riverside Library and is on Birch Ave
The Village Market forms an alternate interior angle with Hilltop Fire Station and is on Pinecrest Street	The Horizon Fitness Center forms a corresponding angle with the Village Market and is on Birch Ave	Northside Pharmacy forms an alternate exterior angle with Horizon Fitness Center and is on Birch Ave
Central Dog Park forms an alternate interior angle with Northside Pharmacy	Evergreen Flower Shop forms a corresponding angle with Central Dog Park	Horizon Medical Clinic Forms an alternate exterior angle with Evergreen Flower Shop
The Mountainview Playground forms a same-side exterior angle with the Horizon Medical Clinic	Central Post Office forms a corresponding angle with the Mountainview Playground	The Tool Shed forms a vertical angle with the Central Post Office
The Oaks Apartment Complex forms a same-side exterior angle with the Tool Shed		

Iron Waste

The names of the locations were generated by the following link:
https://chatgpt.com/share/7f8450f6-1bc4-4ae2-a499-7b858cc61e20

The Rupture Arcade	Crater Market	Shatter Vault
The Coil Refinery	Warden Tower	The Scar Depot
Null Complex	The Fracture Café	Echo Chambers
Dust Square	The Rust Quarry	Gutterworks Shelter
Forge Station	The Wreckage Diner	Obsidian Library
Graveyard Alley		

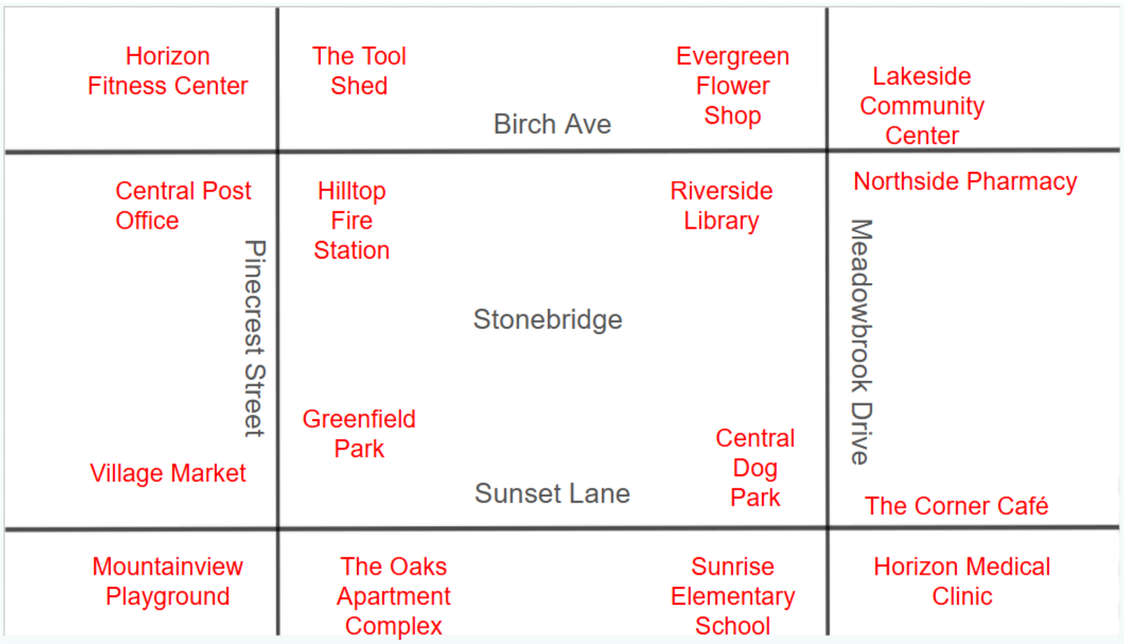
The Rupture Arcade is located on the Northwest corner of Ashen Street and Furnace Lane	Crater Market forms a same-side exterior with the Rupture Arcade and is Bleakway Ave	Shatter Vault forms an Alternate exterior angle with Crater Market and is on Decay Drive
The Coil Refinery forms an Alternate interior angle with the Shatter Vault	Warden Tower forms a same-side interior angle with the Coil Refinery and is on Furnace Lane	The Scar Depot forms an alternate interior angle with Warden Tower and is on Decay Drive
The Null Complex forms a corresponding angle with the Scar Depot and is on Bleakway Ave	The Fracture Café forms a vertical angle with the Null Complex	Echo Chambers forms an alternate exterior angle with The Fracture Café
Dust Square forms a vertical angle with Echo Chambers	The Rust Quarry forms a same-side exterior angle with Dust Square	Gutterworks Shelter forms a corresponding angle with the Rust Quarry and is on Bleakway Ave
Forge Station forms an alternate interior angle with the Gutterworks Shelter	The Wreckage Diner forms an alternate exterior angle with the Forge Station	The Obsidian Library forms an alternate exterior angle with the Wreckage Diner
The Graveyard Alley forms a same-side interior angle with the Obsidian Library		

City Game Answer Keys

This section of **Appendix A** shows the answer key to the city game.

Figure A.3

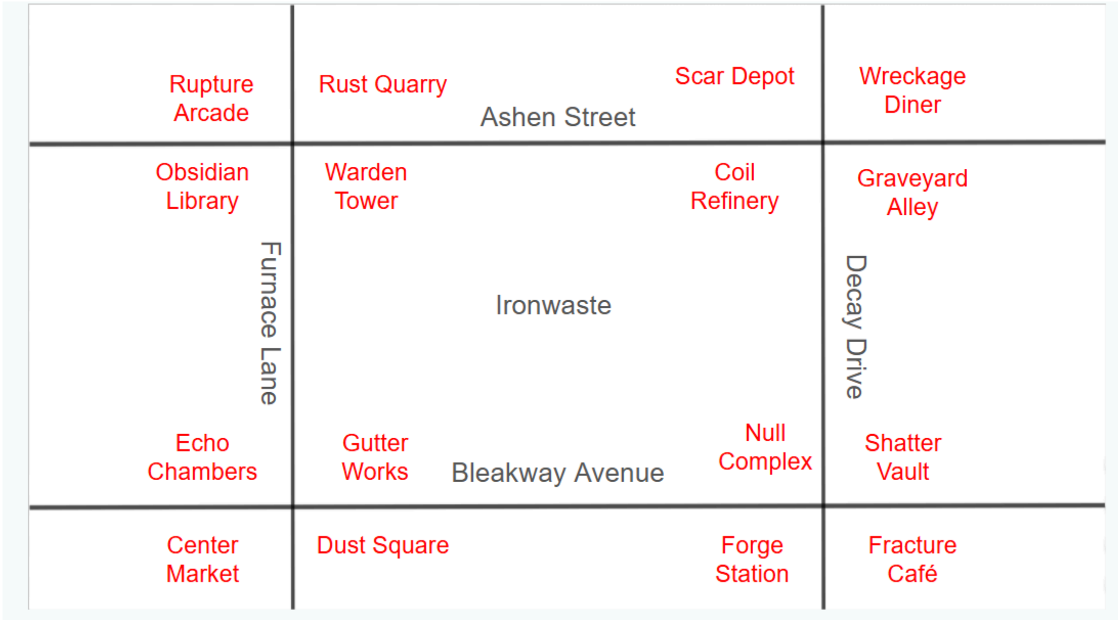
Answer Key for Stonebridge



Note. This figure shows the answer key for where each location should be placed in Stonebridge.

Figure A.3

Answer Key for Stonebridge



Note. This figure shows the answer key for where each location should be placed in Stonebridge.

Appendix B: Car Project Materials

Student Handout

This section of **Appendix B** shows the handout of the Car Project that students will receive.

Buying a Car Project:

In this project, you will explore the math behind purchasing a car so that you are well-informed when you need to buy a vehicle. To complete this project, you will follow the steps below. Please use Google Sheets to complete your calculations, and label your work so it is clear which work goes with each problem. **Answers without supporting work on a Google sheet will receive LITTLE CREDIT.** You may work with a partner for part 1, but you must turn in your own Google sheet. Part 2 must be completed individually.

Part 1: Creating the Model for One Vehicle:

Suppose John wants to purchase a vehicle that has a sticker price of $30,000. He wants to make a 10% downpayment, and he can afford a monthly payment of $500. The bank will give him a loan with a 6% yearly interest rate. We will create a Google sheet to model the amount of money he owes over time. Name your sheet: Part 1.

1. In your google sheet, label one column “variable names” and the column next to it “variable numbers.” List the following variables:
    a. Sticker Price
    b. Downpayment percentage
    c. Yearly interest Rate.
    d. Monthly Payment
2. Now, use these variables to calculate the following items. List your answers as variables in the variable columns.
    a. How many dollars will John put down on the vehicle?
    b. How much will John borrow from the bank? (This is called the principal amount of the loan).
    c. What is the monthly interest rate?
    d. How much interest will John pay in the first month?
3. Now we will create a model to determine how long it takes to pay off the loan of the vehicle.
    a. Label a column “Months,” and make each entry a month number. (Row 2 should have the number 1, row 3 should have the number 2, etc).
    b. Label the next column “Balance Before Payment.” In the first month, this is the same number that you calculated in 2c. To grab this value from the “Variable Numbers” column, you should type “=” and then select the cell that has the number you calculated in 2C.
    c. Label the next column “Monthly payment.” This will be the same number each month, so simply reference the appropriate cell from the variable

numbers column. Use dollar signs to lock this value in. This means that the same cell will still be referenced when we drag this value down later.

d. Label the next column "Interest Paid." Calculate the interest paid by referencing the balance before the payment column.
e. Label the next column principle paid, and calculate this for the first month.
f. Label the next column "Balance After payment." Calculate the balance after the payment in the first month by referencing the appropriate cells.
g. In the balance before payment column and the month 2 row, calculate what the balance before the payment is in the second month.
h. Now we can drag the values of each column down to model what happens over time to the balance of the loan. If your model does not look like it is functioning correctly after the first row, look in the month 3 row. Double-click on cells to check whether or not the correct cells are being referenced for the calculation. If the wrong cells are referenced, then you will need to add or remove dollar signs and drag the cells down again.

4. Once your model is functioning correctly, answer the following questions
    a. How many months does it take to pay off the loan?

    b. How much interest will you pay for the vehicle? Give your answer in total dollars.

    c. What is the total amount of money you will pay for the vehicle?

5. Before moving on further, check your answers with your teacher.

6. What happens if you increase the monthly payment by 10%? Calculate the number of months it takes to pay off the loan, the total amount of money John will pay for the vehicle, and the total amount of interest John will pay.

HINT: Instead of recreating the model again for this new parameter, we can copy and paste our work into a new sheet. Then, we can change our monthly payment, and Google Sheets will do the rest of the work for us. Isn't that much nicer than redoing all of the calculations?

7. What happens if John decreases the monthly payment to $130 per month? Calculate the number of months it takes to pay off the loan, the total amount of money John will pay for the vehicle, and the total amount of interest John will pay. Describe what happens to each of these items as the monthly payment decreases heavily.

8. If the monthly payment goes back to $500, what happens if John makes a 20% downpayment? Calculate the number of months it takes to pay off the loan, the total amount of money John will pay for the vehicle, and the total amount of interest John will pay. Describe what happens to each of these items as the down payment increases.

9. If the monthly payment goes back to $500, what happens if John makes a 5% downpayment? Calculate the number of months it takes to pay off the loan, the total amount of money John will pay for the vehicle, and the total amount of interest John will pay. Describe what happens to each of these items as the down payment decreases.

10. If the downpayment percentage goes back down to 10%, what happens if John gets a different interest rate? Use a 5% interest rate and calculate the monthly payment, the total amount of money you will pay for the vehicle, and the total amount of interest you will pay. Repeat this process for a 7% interest rate. Describe what happens to each of these items as the interest rates vary.

Part 2: Finding your dream vehicle: Complete on your own.

1. Find a car listing online that interests you. Save the link to this posting and take a screenshot of the posting in case the vehicle is sold, and the posting is removed. List the vehicle's make, model, mileage, and price.

2. Let's repeat the modeling process we used with John on your vehicle. Start by creating a new sheet with a clear name such as "Dream Vehicle." Copy and paste

the variable names and variable numbers columns from the previous example. Now let's redefine our variables.

   a. Use the sticker price from your online listing.
   b. Pick a downpayment percentage from the following options: 5%, 10%, 15%, 20%, 25% keeping in mind your findings on questions 8 and 9 of part 1.
   c. Let's use a 6% yearly interest rate. Add this variable to your list. Calculate the monthly interest rate, and add this variable to your list.
   d. Select a monthly payment keeping in mind your findings on questions 6 and 7 in part 1.

3. Now calculate the following items and list them here:
   a. Calculate the downpayment in dollars you will make on your vehicle.
   b. Calculate the amount of money you will borrow from the bank.
   c. Calculate the amount of interest that you will pay in the first month.
4. Now recreate the model created in part 1 question 3.
5. Based on this model, answer the following questions"
   a. How many months does it take to pay off the loan?

   b. How much interest will you pay for the vehicle?

   c. What is the total amount of money you will pay for the vehicle?

6. Once again, let's see what happens if we change the monthly payment.
   a. Select two different monthly payments. Calculate the number of months it takes to pay off the loan, the total amount of money you will pay for the vehicle, and the total amount of interest you will pay. Determine which monthly payment you would want to make if purchasing this vehicle.

7. Now that we have an idea of how much it would cost to acquire the vehicle, let's determine how much it would cost to actually drive the vehicle. Look at your car's listing and find the vehicle's miles per gallon, which is how many miles the car can drive per gallon. If you cannot find this on the car's listing, do a Google search of your vehicle to get an estimate. Write your vehicle's MPG here.
   HINT: there may be one number listed for highway driving and one listed for city driving. If so, make a guess as to whether you will do more highway or city driving, and use that number.

8. Research which grade or type of gas your vehicle takes: Options include Regular (Unleaded), Mid-Grade, Premium, and Diesel. Then research the cost of that one gallon of gas is in your area. Record those items here.

9. Determine the cost per mile to drive your vehicle.

10. Let's suppose you drive this vehicle for 75,000 miles. Assuming the cost of gas stays the same (which it does not in the real world), determine how much total money you would spend on gas over 75,000 miles.

11. Suppose this vehicle had a different grade of gas. Repeat steps 8-10 and describe what the impact of a different grade of gas was.

12. Suppose this vehicle had a 25% increase in its MPG. Repeat steps 8-10 and describe what the impact of this change would be.

13. Research another vehicle of your dreams. Repeat questions 1-10 for this vehicle. Make sure you have a spreadsheet to model this vehicle and you have the following items listed:
    a. A screenshot of the listing
    b. The vehicles make, model, mileage, and price
    c. Pick a downpayment from question 2b in part 2
    d. Pick an interest rate: Use either a 5, 6, or 7% interest rate.
    e. Select a monthly payment.
    f. The number of months it takes to pay off the vehicle
    g. The total amount of interest you pay
    h. The total amount you pay for the vehicle
    i. The vehicle's MPG
    j. The vehicle's grade of gas and cost of that gas in dollars per gallon
    k. The cost per mile to drive the vehicle
    l. The cost of gas for driving this vehicle for 75,000 miles.

14. Now, it is time to choose which vehicle you want to purchase. Provide a 1-2 paragraph justification for your purchase, citing 3 reasons from your work above.

15. Finally, in complete sentences, write three important takeaways from this project that you will consider when purchasing your first vehicle.

Teacher Example Handout

In this project, you will explore the math behind purchasing a car so that you are well-informed when you need to buy a vehicle. To complete this project, you will follow the steps below. Please use Google Sheets to complete your calculations, and label your work so it is clear which work goes with each problem. **Answers without supporting work on a Google sheet will receive LITTLE CREDIT.** You may work with a partner for part 1, but you must turn in your own google sheet. Part 2 must be completed individually.

Part 1: Creating the Model for One Vehicle:

Suppose John wants to purchase a vehicle that has a sticker price of $30,000. He wants to make a 10% down payment, and he can afford a monthly payment of $500. The bank will give him a loan with a 6% yearly interest rate. We will create a Google sheet to model the amount of money he owes over time. Name your sheet: Part 1.

1. In your google sheet, label one column "variable names" and the column next to it "variable numbers." List the following variables:
    a. Sticker Price
    b. Downpayment percentage
    c. Yearly interest Rate.
    d. Monthly Payment
2. Now, use these variables to calculate the following items. List your answers as variables in the variable columns.
    a. How many dollars will John put down on the vehicle?
    b. How much will John borrow from the bank? (This is called the principal amount of the loan).
    c. What is the monthly interest rate?
    d. How much interest will John pay in the first month?
3. Now we will create a model to determine how long it takes to pay off the loan of the vehicle.
    a. Label a column "Months," and make each entry a month number. (Row 2 should have the number 1, row 3 should have the number 2, etc).
    b. Label the next column "Balance Before Payment." In the first month, this is the same number that you calculated in 2c. To grab this value from the "Variable Numbers" column, you should type "=" and then select the cell that has the number you calculated in 2C.
    c. Label the next column "Monthly payment." This will be the same number each month, so simply reference the appropriate cell from the variable

numbers column. Use dollar signs to lock this value in. This means that the same cell will still be referenced when we drag this value down later.

d. Label the next column "Interest Paid." Calculate the interest paid by referencing the balance before the payment column.
e. Label the next column principle paid, and calculate this for the first month.
f. Label the next column "Balance After payment" Calculate the balance after the payment in the first month by referencing the appropriate cells.
g. In the balance before payment column and the month 2 row, calculate what the balance before the payment is in the second month.
h. Now we can drag the values of each column down to model what happens over time to the balance of the loan. If your model does not look like it is functioning correctly after the first row, look in the month 3 row. Double-click on cells to check whether or not the correct cells are being referenced for the calculation. If the wrong cells are referenced, then you will need to add or remove dollar signs and drag the cells down again.

4. Once your model is functioning correctly, answer the following questions
   a. How many months does it take to pay off the loan?

64 months

   b. How much interest will you pay for the vehicle? Clearly label where you calculate this item in your google sheet.

Interest paid is approximately $4549.55. Give leeway within a dollar if students highlight an additional cell. Potential mistake is dragging the cells down for longer than 64 months, and summing all of the interest, including negatives after 64 months.

   c. What is the total amount of money you will pay for the vehicle? Clearly label where you calculate this item in your google sheet.

   The total amount paid is $31,549.55.

5. Before moving on further, check your answers with your teacher.

6. What happens if you increase the monthly payment by 10%? Calculate the number of months it takes to pay off the loan, the total amount of money John will pay for the vehicle, and the total amount of interest John will pay.

HINT: Instead of recreating the model again for this new parameter, we can copy and paste our work into a new sheet. Then, we can change our monthly payment, and Google Sheets will do the rest of the work for us. Isn't that much nicer than redoing all of the calculations?

The new loan term is 57 months. The interest paid is $4058.10. The total amount paid is $31,058.

7. What happens if John decreases the monthly payment to $130 per month? Calculate the number of months it takes to pay off the loan, the total amount of money John will pay for the vehicle, and the total amount of interest John will pay. Describe what happens to each of these items as the monthly payment decreases heavily.

After 200 months, the loan will still not be paid off. Students should observe that the balance of the loan is increasing, and will never be paid off at this payment. Students should come to the conclusion that the monthly payment must be higher than the interest paid to pay off the loan.

8. If the monthly payment goes back to $500, what happens if John makes a 20% downpayment? Calculate the number of months it takes to pay off the loan, the total amount of money John will pay for the vehicle, and the total amount of interest John will pay. Describe what happens to each of these items as the down payment increases.

The new loan term is 55 months. The total interest paid is $3512.21 and the total amount paid is $27,512. As the down payment amount increases, the loan term, interest paid, and total amount paid all decrease.

9. If the monthly payment goes back to $500, what happens if John makes a 5% downpayment? Calculate the number of months it takes to pay off the loan, the total amount of money John will pay for the vehicle, and the total amount of interest John will pay. Describe what happens to each of these items as the down payment decreases.

The new loan term is between 68 months. The total interest paid is $5135.31, and the total amount paid is $33,631. As the down payment decreases, the loan term, interest paid, and total amount paid all increase.

10. If the downpayment percentage goes back down to 10%, what happens if John gets a different interest rate? Use a 5% interest rate and calculate the monthly payment, the total amount of money you will pay for the vehicle, and the total amount of interest you will pay. Repeat this process for a 7% interest rate. Describe what happens to each of these items as the interest rates vary.

At 5%, the new loan term is 62 months. The total interest paid is $3650.97 and the total amount paid is $30,650.97. At 7%, the new loan term is 66 months. The total interest paid is $5523.39 and the total amount paid is $32,523.40. As the interest rate decreases, the loan term, total interest and total amount paid all decrease.

Part 2: Finding your dream vehicle: Complete on your own.

1. Find a car listing online that interests you. Save the link to this posting and take a screenshot of the posting in case the vehicle is sold, and the posting is removed. List the vehicle's make, model, mileage, and price.

All items must be included. Answers will vary depending on the choice of vehicle for all of Part 2.

Teacher Example Spreadsheet

This section of **Appendix B** is a placeholder for the teacher example spreadsheet, which can be found on the Excel sheet labeled Car_Project_Teacher.

Rubric

This section of **Appendix B** shows the rubric that can be used to assess the car project.

Part 1	0 pts	2 pts	3 pts	4.5 pts	5 pts
Questions 1 and 2	No work was submitted.	Over half of the variables are missing or have major errors made in calculations.	Multiple items are missing or major errors are made in calculations.	All 8 items are listed and calculated with minor errors.	All 8 items are listed and calculated correctly.
Question 3	No work was submitted.	An attempt was made to make the model, but there are major errors AND missing parts.	The model has multiple major errors OR missing parts.	All parts of the model are completed and calculated with minor errors.	All parts of the model are completed and calculated correctly.
Question 4	No work was submitted.	An attempt was made, but no answers were correct.	Multiple questions had major errors.	All 3 questions are answered with minor errors.	All 3 questions are answered correctly.
Questions 6 and 7	No work was submitted.	An attempt was made, but no answers were correct.	Major errors were made in answering the questions or creating the model.	Minor errors were made in answering the questions or creating the model.	All 3 questions are answered correctly and supported with a correct model.
Questions 8 and 9	No work was submitted.	An attempt was made, but no	Major errors were made in	Minor errors were made in	All 3 questions are

		answers were correct.	answering the questions or creating the model.	answering the questions or creating the model.	answered correctly and supported with a correct model.
Question 10	No work was submitted.	An attempt was made, but no answers were correct.	Major errors were made in answering the questions or creating the model.	Minor errors were made in answering the questions or creating the model.	All 3 questions are answered correctly and supported with a correct model.

Rubric: Part 2. This rubric will be applied to each vehicle, so you will be assessed on this rubric twice for questions 1-12, but only once for questions 14 and 15.

	0 pts	2 pts	3 pts	4.5 pts	5 pts
Questions 1, 2, and 3	No work was submitted.	Over half of the items are missing or have major errors made in calculations.	Multiple items are missing or major errors are made in calculations.	All items are listed and calculated with minor errors or one item is not listed.	All items are listed and calculated correctly.
Question 4	No work was submitted.	An attempt was made to make the model, but there are major errors AND missing parts.	The model has multiple major errors OR missing parts.	All parts of the model are completed and calculated with minor errors.	All parts of the model are completed and calculated correctly.
Question 5	No work was submitted.	An attempt was made, but no answers	Multiple questions had major errors.	All 3 questions are answered	All 3 questions are answered

		were correct.		with minor errors.	correctly.
Question 6	No work was submitted.	An attempt was made, but no answers were correct.	Major errors were made in answering the questions or creating the model.	Minor errors were made in answering the questions or creating the model.	All 3 questions are answered correctly and supported with a correct model.
Questions 7, 8, 9, and 10	No work was submitted.	An attempt was made, but no answers were correct.	Major errors were made in answering the questions.	Minor errors were made in answering the questions.	All 4 questions were answered correctly and were supported with work or sources.
Question 11	No work was submitted.	An attempt was made, but no answers were correct.	Major errors were made in answering the questions.	Minor errors were made in answering the questions.	All 4 questions were answered correctly and were supported with work or sources.
Question 12	No work was submitted.	An attempt was made, but no answers were correct.	Major errors were made in answering the questions.	Minor errors were made in answering the questions.	All 4 questions were answered correctly and were supported with work or sources.
Question 14	No work was submitted.	The justification is not 1-2	The justification is not 1-2	The justification is 1-2	The justification is 1-2

		paragraphs long AND only 1 reason is cited from their previous work.	paragraphs long OR only 1 reason is cited from their previous work.	paragraphs long, and 2 reasons are cited from their previous work or there are minor flaws in their reasoning	paragraphs long, and 3 reasons are cited from their previous work.
Question 15	No work was submitted.	An attempt was made but no complete sentences were used or the takeaway does not relate to the project.	Only 1 takeaway was listed in complete sentences.	2 takeaways are listed in complete sentences OR 3 takeaways are listed but not in complete sentences.	3 takeaways are listed in complete sentences.

Appendix C: Discovering Slope Materials

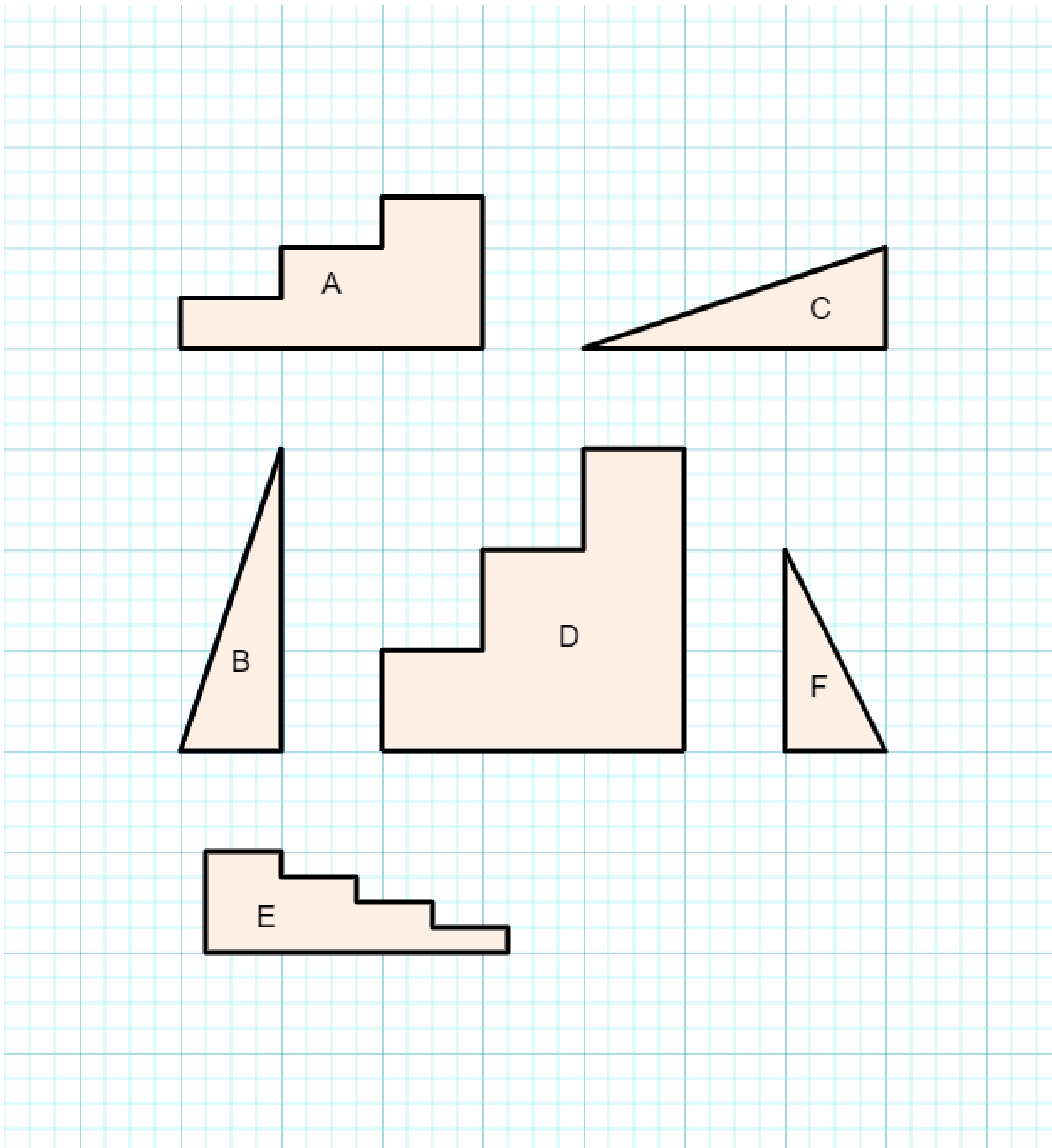
Student Worksheet

This section of **Appendix C** shows. the worksheet required for the slope activity.

Slope:

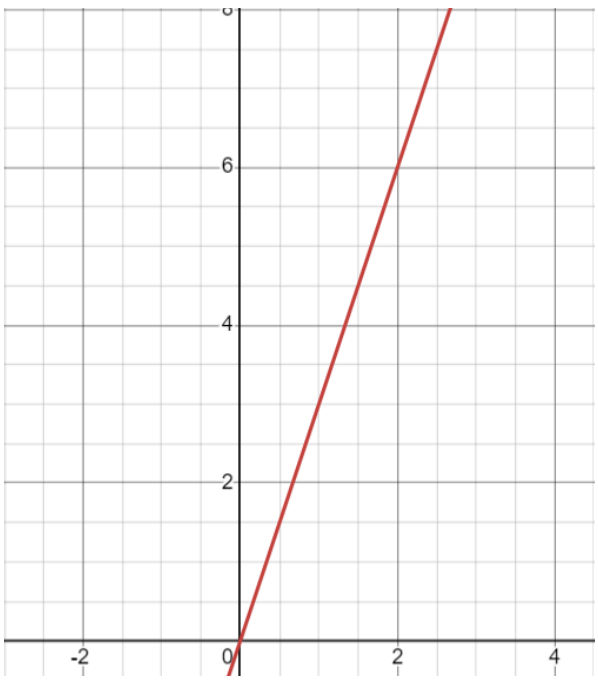
Name: ___________________

Order the ramps and staircases by steepness. Give numerical values to the steepness of each ramp. Give 4 complete sentences explaining your reasoning.



In Class Practice:

1.



2.

Teacher Slideshow

This section of **Appendix C** shows screenshots of the slides in the slideshow.

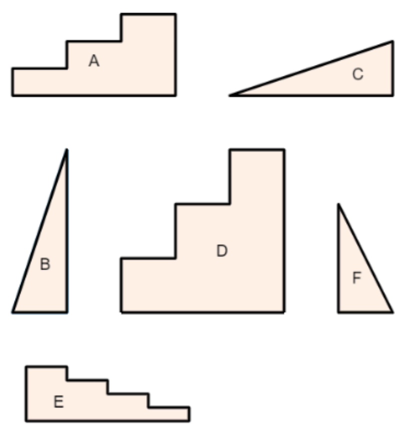
Slide 1

Discovering Rate of Change

Slide 2

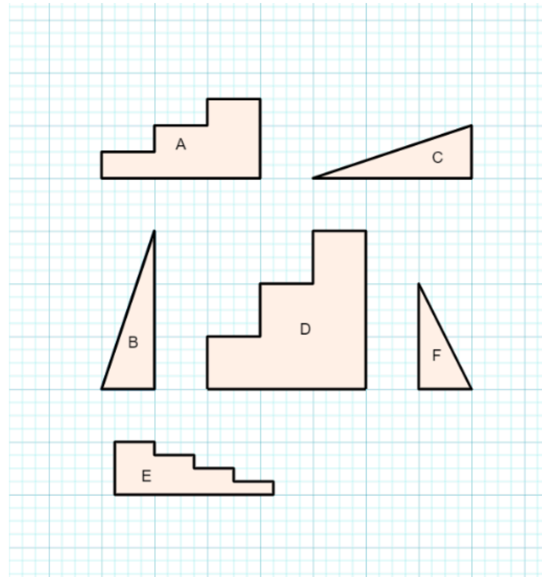
Order the ramps by steepness. Which one is the easiest to climb and which one is the hardest?

Order the staircases by steepness. Which one is the easiest to climb and which one is the hardest?



Slide 3

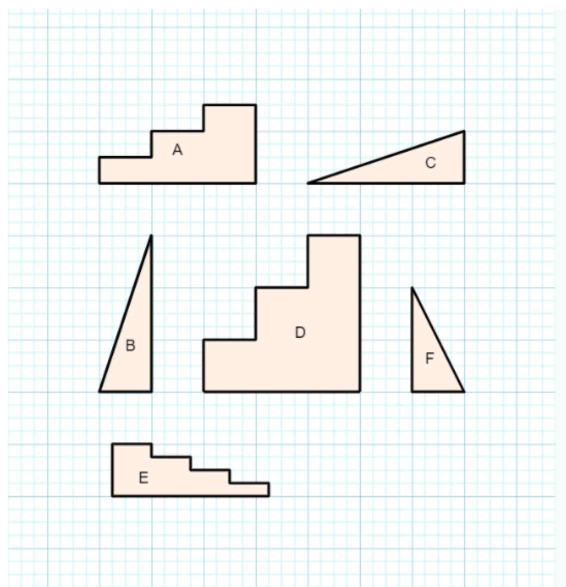
If we add a grid, can we assign numbers to the steepness of each ramp?



Slide 4

Counting as a Strategy

How do we report our answer?



Slide 5

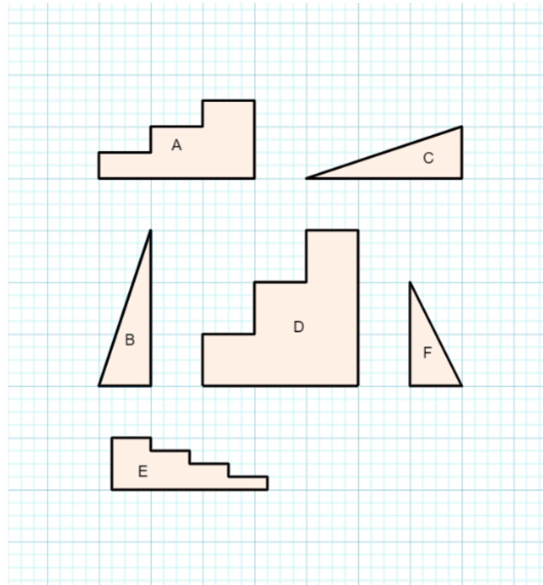
Counting as a Strategy

How do we report our answer?

Should it be units counted up (rise) over units counted sideways (run) OR

Should it be units counted sideways over units counted up?

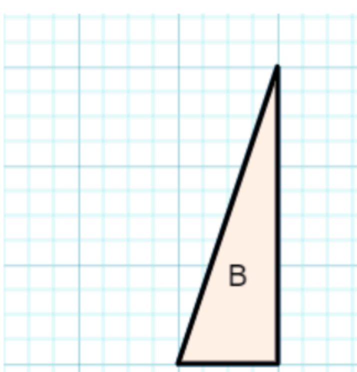
Discuss this question in groups. Come up with a piece of evidence to support your reasoning.

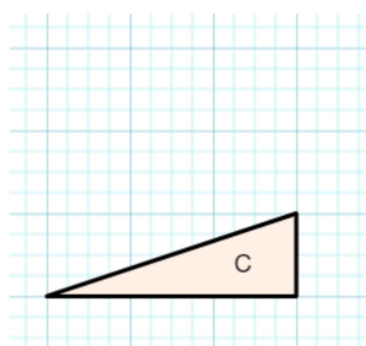


Slide 6

Compare B and C

Calculate steepness for B and C using both rise divided run and run divided by rise





Slide 7

Rise over Run

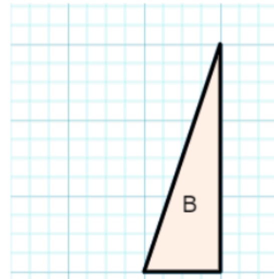
B: 15⁄5= 3

C: 5⁄15 = ⅓

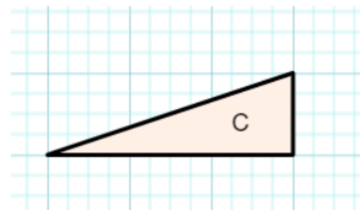
Run over Rise

B: 5⁄15 = ⅓

C: 15⁄5= 3

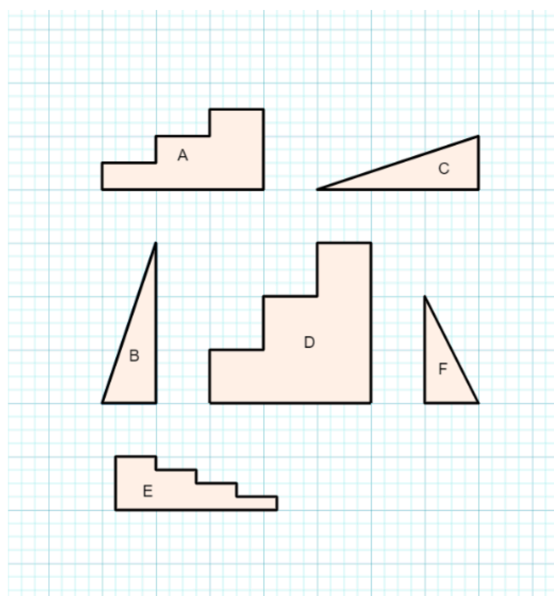


So which one should we use? Why?



Slide 8

Does the direction of the ramps or staircases matter?



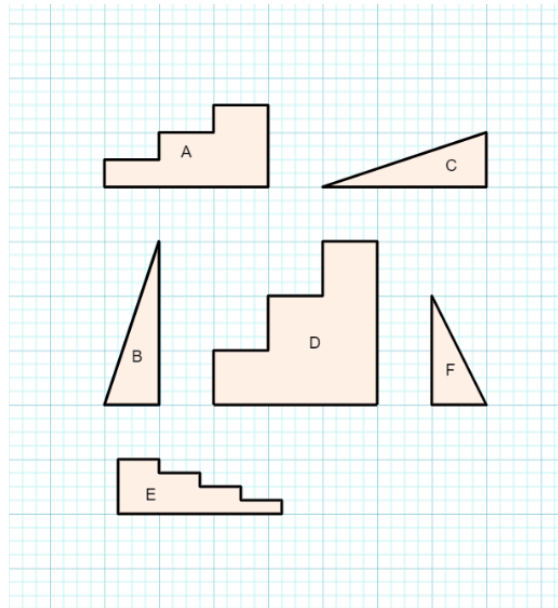
Slide 9

Does the direction of the ramps or staircases matter?

If you were a person walking from left to right on the graph paper, would you rather walk on ramp B or ramp F ?

What if you were walking from right to left?

Which is easier to visualize?



Slide 10

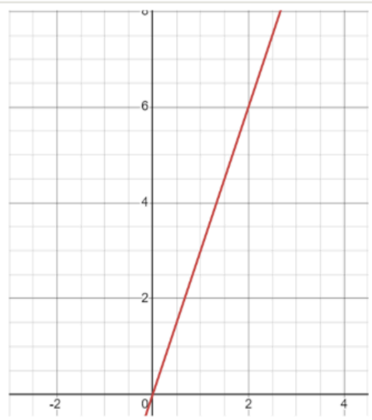
Conclusion

When we read a book, we read left to right. So for consistency, when reading graphs, we will also read left to right.

Slide 11

Let's practice with our new knowledge

1.



2.