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The Relations Between Mathematics and Philosophy

by

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Submitted in partial fulfillment of the requirements for the degree of Bachelor of Arts in the Department of Mathematics, Carroll College, Helena, Montana

June 3, 1934
Born--Conrad, Montana--September 13, 1912
Cascade Public School--Cascade, Montana--1918-20
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Outline of Thesis:

Introduction:
A. Definition of terms
   a. Definition of mathematics
   b. Definition of philosophy
B. Outline of thesis
   a. Extent and Object
   b. Plan of procedure

I. Mathematics and Philosophy
A. Validity of Mathematics
   a. Metaphysics necessary
   b. This necessity not always formal
B. Example of this necessity
   a. Practical explanation of this necessity
   b. Practical example of this necessity

II. Relationship between Mathematics and Practical Philosophy
A. Ancient times
   a. Concrete example
   b. Why Greeks studied mathematics—truth
   c. Greek ideas with practical applications today
B. Present time
   a. Scientific and material progress
   b. Political, economic, and social progress
III-How Mathematics is in itself a Philosophy

A. How it justifies its right of existence
   a- Its place in the world-scheme of knowledge

B. Attractiveness of Mathematics
   a- self-sufficiency
   b- utility
   c- long and glorious history
      1. Pythagoras
      2. Plato
      3. Descartes

Conclusion:
"The history of mathematics is one of the large windows through which the philosophic eye looks into past ages and traces the line of intellectual development." Thus does a prominent mathematician characterize the general relationship between the science of mathematics and the science of philosophy.

Before I proceed further, it would perhaps be well to stop and consider the method and viewpoint under which this topic will be treated. I will, therefore, define terms to determine upon which premises these relationships are to be built.

Definition of Philosophy: "Lit., the love of wisdom; in actual usage, the knowledge of phenomena as explained by, and resolved into causes and reasons, powers and laws.—In more general application, and usually with 'the' or 'a', philosophy denotes a systematic body of general conceptions.—In the broadest scope, any attempt to present or conceive a systematic view of all things is a philosophy."^2 Thus by the last part of this definition when one speaks of a person's philosophy of life, one is not referring to the formal science of philosophy, but to this individual's ideas and ideals about which the pattern of his life is woven. However, unless otherwise

2. Webster's Dictionary------ —Page 1631
stated, the term "philosophy" as used here will be used in the strict sense.

Definition of Mathematics: "That science, or class of sciences, which treats of the exact relations existing between quantities or magnitudes and operations, and of the methods by which, in accordance with these relations, quantities sought are deducible from others known or supposed; the science of serial, spatial, quantitative, and magnitudinal relations; the science of order." 3

With such limited space, I will be able to treat with this subject only in a very general way. Were I to treat of each branch of philosophy, volumes could be written and yet not exhaust the subject. Therefore, I will attempt only to give the reader an idea of the importance of this general relationship and give him a background from which he can proceed to develop in his own mind and in his own way a closer connection between these two all-important branches of thought. I use the word "all-important" advisedly, because, as I will show, no other science or intellectual activity of man could exist were it not for the principles, both abstract and concrete, that are taken from both mathematics and philosophy. Without philosophy; chemistry, physics, biology, etc., and even mathematics would

3. Webster's Dictionary----------Page 1330
be meaningless systems of symbols and relationships whose truth or practicability we could not know. Mathematics establishes those relationships to which philosophy gives a meaning, and which are concretely applied in these various other sciences.

In showing the relationships between mathematics and philosophy, I will employ a three-fold scheme. First, I will show how philosophy makes of mathematics a true and a practical science; second, I will show the definite relationship between mathematics and philosophy; and third, I will show how mathematics is in itself a philosophy. In each of these outlined plans I have used the word "philosophy" with slightly different connotations, and as I treat each plan, I will show exactly what I mean by philosophy as I use it there.

Without philosophy, mathematics would be a system of symbols and relationships utterly useless and void of meaning. By philosophy, here, I mean that particular branch of philosophy, metaphysics, which proves the reality of all being and of all existence. We study facts but do not get the unity behind them. Metaphysics gives us this unity. Not satisfied with mere appearances as an explanation of things, we seek to learn the relationships and meanings of all objects by the study of metaphysics.
I do not mean that all who make use of mathematics (and a very large percent of people do to some extent) know of and understand the metaphysical principles behind mathematics. In fact, the vast majority of these people have never even heard of metaphysics, but tho they do not formally apply the principles, they do unknowingly apply them. But it is an assured fact, that all the great mathematicians have seen the philosophic significance behind the mathematical facts that they expound.

Let us look at a rudimentary example of this philosophic significance. The mathematician says that 2+2=4. In fact, we all say that 2+2=4. Could our belief and the belief of the mathematician in the validity of this statement be shaken, the science of mathematics could not exist. But what is the concept of 2 and of 4 and of plus that so establishes this definite relationship? The idea of being 2 and of being 4 and of their necessary relationship is from metaphysics whether one formally recognizes the fact or not. The idea of 2 and of 4 is from the essence of 2 and of 4. The essence 2 plus the essence 2 equals the essence 4. In themselves 2 and 4 are symbols which mean nothing. The Romans wrote II and IV, and they meant the same thing. 2 and 4 mean nothing in themselves, except, that arbitrarily, we say these symbols are a representation of a true essence. Essence is from metaphysics and the mathematician
can know nothing of it except through metaphysics. Our question here of essence is a question of the content of idea. This is metaphysics.

Again the mathematician says: since \(a=b\) and \(b=c\); then necessarily \(a=c\). This is a fundamental relationship in mathematics that two things equal to a third are equal to each other. The casual mathematician would say that this fact is self-evident, but on the basis of mathematics alone he can not prove this self-evident relationship. He must turn to logic and to metaphysics for the proof. He proves it by taking from logic the law of identity through a middle term; the principle that two things like unto a third are like unto each other. But this is an axiom in logic. This axiom rests upon the metaphysical principle of identity; that a thing is identical with itself. Pure mathematics is merely a branch of metaphysics. Mathematics is but the philosophy of quantity. \(x+y+z\) is a purely arbitrary symbolism used in order to represent an idea (a real quantity) in the mind. Pure mathematics is no more or less than a study of the essence of quantity, and we must turn to metaphysics to see what quantity really is.

Countless examples showing the dependence of mathematics upon metaphysics might be cited, but these few should be sufficient to illustrate the point. These mathematical prin-
ciples are fundamental to mathematics. Unless these two and other equally fundamental principles can be proven, the science of mathematics means nothing. It would be an empty set of rules of procedure whose truth or practicability could not be proven. As I have shown in the above two examples these principles cannot be proven in mathematics but must be proven by metaphysics; thus is seen the absolute necessity of philosophy as a basis for mathematics.

Now I will show the definite relationship existing between mathematics and philosophy. Philosophy here means life. As a practical example I will quote from Cajori. In writing of a few simple geometrical principles he says: "The Egyptians must have made use of the above theorems on the straight line in some of their constructions found in Ahmes papyrus, but it was left for the Greek philosopher to give these truths, which others saw, but did not formulate into words, an explicit, abstract expression, and to put into scientific language and subject to proof that which others merely felt to be true." 4

In the above paragraph we saw that the Egyptians made use of certain elementary theorems without formally developing them in the abstract. Likewise in the case of other constructions, such as the pyramids, the Egyptians must have made use of these few elementary 4. "A History of Mathematics"--Cajori--Page 16
mathematical principles. But we can truthfully say in regard to more advanced mathematics that mathematical relationships and concepts have usually been completely developed before there was any practical application of these concepts. In this, mathematics merely follows the ordered rules of thought. Before we can do anything, we must first, if we are to proceed logically— and how else can we proceed except by trial and error—which should be a last resort— have principles to go on. These at first will necessarily be abstract.

Again let us look at ancient ideas of mathematics and the use made of these ideas. "Two and a half thousand years ago— Thales of Miletus measured the heights of the pyramids by their shadows and predicted the solar eclipse of 585 B.C.," 5 Here we see an early mathematician who made practical use of his knowledge, but we also see that he was dealing with only primary principles of geometry. Further we see: "Thales traveled in Egypt and no doubt was familiar with the empirical mathematics deduced by the Egyptians and used to build the pyramids and to fix the boundaries of the Nilean farms, and it is very significant that, while their rules deduced by observation were for the most part inaccurate, those deduced by Thales' intellect are still valid after the tests of twenty-five centuries." 6 This quotation is not in contradiction to the statement made previously (that mathematics is first abstract, then

5. "Philosophy of Mathematics"—Shaw—Page 35
6. Ibid
concrete). The previous statement was to the effect that higher mathematics is abstract long before any concrete use of it is seen.

Above, we see the Egyptians deducing mathematical laws from their practical experience, and we see Thales correcting these laws so that they stand today. We must first of all remember that these plans were very elementary, and we must consider the fact that in all probability a few first principles were conceived in the minds of the Egyptians before they started their constructions. They did not blindly begin to lay stone upon stone, and then, when their whole structure was finished (by blind design), deduce mathematical concepts from their observations of the structure. It would be extremely foolish to think that their marvelous constructions were brought about just by their chance laying of stones. History shows us that the Egyptians built according to well-formulated plans. That they didn't see or that they didn't develop the mathematical significance of these plans does not mean that these plans were not worked out abstractly in their minds before these plans were concretely applied. And the more developed the mathematics the slower will be its concrete application.

We are only just beginning to make real applications of many of the mathematical laws that were first studied by the early Greeks. They did not study them because they foresaw their marvelous use in the centuries to come; they were interested in the relationships which they found to exist. These early Greeks were searching
primarily for truth; so, it is easy to see why they turned to the study of mathematics. They saw the universal and eternal truths of mathematics. They saw the relationships which fitted naturally and logically into the whole. This was the reason they studied mathematics. Later I will show how many of these Greek thinkers, seeing the unchanging truths of mathematical relationships, based their philosophies upon mathematics.

We see that the Greeks studied geometry and had some knowledge of conic sections two thousand years before this knowledge was used. It is practical geometry that "that makes the modern giant steel structure of many stories secure." But let us see how more advanced theories have direct application. For instance, imaginary and complex functions were developed long before they were of use to the wireless telegrapher. Again, "For example, in the shadows of an electric lamp we may see the theory of bilinear quadratics, and the nets of orthogonal curves on a surface contain the theory of functions of a complex variable. The many-faced crystal reflects in its facets the theory of groups, and in the dreams of imaginary four-dimensional space we have a perfect picture of electrodynamics, that is to say, certain differential equations." This quotation shows the practical application of certain principles of geometry.

7. Ibid-----Page 36
8. Ibid-----Page 37
We have barely begun to realize the potentialities of the applications of mathematics. One cannot readily see the application of the theories of men like Einstein and Le Mettre. Indeed, one hears the modern cynic say in regard to such theories: "They may be perfectly true but what practical use can they be to the world." Their criticism is not unnatural. Indeed, a similar condition exists in all fields of human thought. Had the thinkers of the world first stopped to see what practical use could be made of their ideas, use to themselves and to society, the material progress of the human race would be slow indeed. In the field of science (which is dominated by mathematics) men have worked out their abstract principles because they were intrigued by the necessary relationships which they found worked with equal truth experimentally. Our leaders in the field of science have always been men who were interested in science because of these truths, not because of the utility of science to themselves. This is the reason that we have made such marvelous material progress, and the reason that we will make greater and continued material progress in the years to come. "Progress means growth of soul, and growth of soul means growth of truth. The more clearly and completely truth is mirrored in a man, the higher he ranges in the scale of evolution." 9

But in our social, political, and economic relations of life,

9. "Philosophy as a Science"—Carus—Page 20
which is our philosophy of life, we have lagged behind. We have not learned how to make use of our material benefits. We have not learned this because in philosophy, unlike in science, at present the utilitarian holds sway; the man whose motives are actuated by their utility towards himself and society. This is the reason that we have not been able to properly control the material progress which we have made. We must more closely unite science and philosophy. Until a close connection exists, practical science will continue to forge ahead of practical philosophy. And as long as science does progress faster, we will not be able to fully enjoy the benefits of science. It would be foolish to slow down science. The correct method is to remodel our philosophy; so that, the necessary relation between the two will again be brought about. It is now that the all-important relationship between mathematics (practical mathematics in the various branches of science) and philosophy (our scheme of life) is brought forcibly to our attention.

Having seen this relationship of mathematics to philosophy, I shall take up the third part of my plan and show now mathematics has in itself a philosophy and see how mathematics justifies its right of existence. As a good illustration of what mathematics means in our world-plan, I will quote as follows: "In the Norse mythology there was an ash tree, Yggdrasil, which supported the
universe. Its three roots were fast in the realms where abode the shades of the dead, the race of man-kind, and the frost giants. Its lofty top was in the heavens, where abode the eagle of wisdom, and in the four corners gamboled four stags. We may well take this symbol as a very fair representation of mathematics—the sequoia that supports the universe of knowledge. It derives its stability from the roots that it sends out into the laws of nature, into the reasoning of men, into the accumulated learning of the dead. Its trunk and branches have been built during the past ages out of the fibers of logic; its foliage is in the atmosphere of abstraction, its inflorescence is the outburst of the living imagination. From its dizzy summit genius takes its flight, and in its wealth of verdure its devotees find an everlasting holiday.  

There are three facts concerning mathematics that make it especially attractive for the thinker. These are: one, its self-sufficiency; two, its utility; three, its long and glorious history. I will show that mathematics has in itself a philosophy; that it is autonomous; it is sufficient in itself. By sufficient I mean that while its validity rests with metaphysics, it (mathematics) after its validity is proven contains in itself all its laws, its nature, and its principles. "Not in philosophy, not in science, not in psychology, not in logic, can we dis-
cover these things, but only in mathematics. It does not yield us transcontental space or time, or the categories of reason. It does not tell us whether physical space is Euclidean, Riemannian or Lobatchevskian. It does not decree the way of a cell of protoplasm or the logarithm of a sensation. It is in itself a living thing, developing according to its own nature, and for its own ends, evolving through the centuries, yet leaving its records more imperishable than the creatures of geology."

This quotation aptly shows the self-sufficiency of mathematics. One likes to deal with a subject that does not depend on the vagrancies of other studies. Mathematics does need metaphysics as a basis for its validity, but once this validity is satisfactorily proven, it (the validity) remains unchanged as the metaphysics it is based upon is as fundamental, unchanging, and self-sufficient as is mathematics itself.

Now I will deal with the second fact of the attractiveness of mathematics, its utility. Its utility in chemistry, in physics, etc., and its every-day applications is so evident that we need dwell no further on this. But its utility in some of the higher and more modern extensions is more difficult to see. As Cajori says: "If it be asked wherein the utility of some modern extensions of mathematics lies, it must be acknowledged that it is at present difficult to see how some of them are ever to become
applicable to questions of common life or physical science. But our inability to do this should not be urged as an argument against the pursuit of such studies. In the first place, we know neither the day nor the hour when these abstract developments will find application in the mechanical arts, in physical science, or in other branches of mathematics. For example, the whole subject of graphical statics, so useful to the practical engineer, was made to rest upon von Staudt's 'Geometrie der Lage'; W. R. Hamilton's 'principle of varying action' has its use in astronomy; complex quantities, general integrals, and general theorems in integration offer advantages in the study of electricity and magnetism. 'The utility of such researches,' said Spottiswoode in 1878, 'can in no case be discounted, or even imagined beforehand. Who, for instance, would have supposed that the calculus of forms or the theory of substitutions would have thrown much light upon ordinary equations; or that Abelian functions and hyperelliptic transcendentals would have told us anything about the properties of curves; or that the calculus of operations would have helped us in any way towards the figure of the earth?'' Thus we see, that while the utility of certain mathematical concepts may not be evident, we can not tell, but that at any time, their utility will become perfectly evident.


Mathematics has a long and glorious history. "The con-
templation of the various steps by which mankind has come into possession of the vast stock of mathematical knowledge can hardly fail to interest the mathematician. He takes pride in the fact that his science, more than any other, is an exact science, and that hardly anything ever done in mathematics has proved to be useless. The chemist smiles at the childish efforts of alchemists, but the mathematician finds the geometry of the Greeks and the arithmetic of the Hindus as useful and admirable as any research of to-day. He is pleased to notice that though, in course of its development, mathematics has had periods of slow growth, yet in the main it has been pre-eminently a progressive science. 13

Mathematics is as old as human thought. Throughout all its shows a continuousness of development and a validity lacking to any other study. Nothing has ever been done in vain in mathematics. Its first developments are as true and useful today as they were thousands of years ago. Mathematics is not a group of varying principles and concepts, of differing opinions. It, as it has always been, is a homogeneous whole. Each new development has merely enlarged or clarified this whole. We can truthfully say that: "We are today heirs of the whole past in mathematics." 14

Pure mathematics is always and universally true. It may err in its conclusions because they are based upon false premises. This is the reason why mathematicians may differ widely. Take the

13. Ibid-------------------------Page 1
case of Einstein and Le Mettre. Each has a very different theory of the universe and each has worked out his theory by mathematics. The difference of their theories is not that the mathematics of one erred; it is that each started with different premises. Which theory, if either, is true depends not upon the mathematics used, but the premises assumed. We can say that given true premises to begin with, and developing them with mathematics and that alone, we cannot err.

As we look over the history of mathematics, we see that many of the great mathematicians were also great philosophers. And their philosophy has been vitally affected by their mathematical concepts. We can not doubt that "the development of mathematics has profoundly influenced philosophy. We need but mention Pythagoras, Plato, Descartes, Spinoza, Leibniz, Kant, Comte, and Russell, in order to call to mind philosophers whose systems were controlled largely by their views of mathematics. There have been also mathematicians who have been at the same time philosophers, and whose criticisms have largely influenced existing systems. The existence indeed of mathematics, its evergreen growth, and its constant success in creating a body of knowledge whose value is universally admitted, are a challenge to the philosopher to do as much, and at the same time an encouragement to him to persist.
in his search for the explanation of things as they are. There is at the present time an increasing trend toward each other of the two disciples. The philosopher is confronted, too, with the added difficulty that he cannot hope to have a complete system unless he accounts for the existence of mathematics and assigns a value to it in human economy, and in order to do this he must perforce learn some mathematics. He must know what the mathematician has found out for himself about his own science, and the significance of what he has found out for the rest of the theory of knowledge."

Let us look briefly at the philosophic views of a few of these philosophers mentioned above paying particular attention as to the manner in which their mathematical concepts influenced their philosophy. One—Pythagoras, who founded the Pythagorean school which taught philosophy, mathematics, and natural science. Pythagoras based his philosophy upon the postulate that number is the cause of the various qualities of matter. This led him to exalt arithmetic, as distinguished from logistic, out of all proportion to its real importance. It also led him to dwell upon the mystic properties of numbers and to consider arithmetic as one of the four degrees of wisdom—arithmetic, music, geometry, and spherics (astronomy), these forming the quadrivium of the Middle Ages. Aristotle tells us that Pythagoras related the virtues to numbers, and Plutarch says that he believed that earth was pro-

15. Ibid-------------------Page 191
duced from the regular hexahedron, fire from the pyramid, air from the octahedron, water from the icosahedron, and the heavenly sphere from the dodecahedron, in all of which the physical elements are related both to number and to form. Philolaus probably voiced the teaching of the master when he asserted that five is the cause of color, six of cold, seven of health, and eight of love." Tho the members of the Pythagorian school erred in their explanation of nature, the substitution of the abstract number for the material principle of the early Ionians marks some progress. They insisted more upon quantity whereas the Ionians insisted upon quality. This school marks an advance in human thought, and their philosophic system shows the influence of their mathematical principles.

Two: Plato like Pythagoras was interested in arithmetic rather than logistic. He says in his "Republic" that arithmetic has a double use, military and philosophical—"'For the man of war must learn the art of numbers or he will not know how to array his troops; and the philosopher also, because he has to arise out of the sea of change and lay hold of true being, and therefore he must be an arithmetician—Arithmetic has a very great and elevating effect, compelling the mind to reason about abstract number.'" We will not deal with Plato's theories of numbers, his geometry, or his philosophy. He was merely quoted

17. Ibid---------------------------Page 89
because in the paragraph from him is seen how closely Plato thought philosophy and mathematics should be connected.

Three; Descartes' name will figure prominently in any History of Philosophy and he did next to Newton, probably the most of any man towards the revolutionizing of mathematics in the 17th Century. A striking statement of the effect of mathematics upon the mind of Descartes is made by John Stuart Mills: "Descartes is the complete test type which history presents of the purely mathematical type of mind—that in which the tendencies produced by mathematical cultivation reign unbalanced and supreme.' Although the statement may well be questioned, it is interesting as a striking assertion if for no other reason." 17a

"Though he professed orthodoxy in faith all his life, yet in science he was a profound sceptic. He found the world's brightest thinkers had been long exercised in metaphysics, yet they had discovered nothing certain; nay, had even flatly contradicted each other. This led him to the gigantic resolution of taking nothing whatever on authority, but of subjecting everything to scrutinous examination, according to new methods of inquiry. The certainty of the conclusions in geometry, and arithmetic brought out in his mind the contrast between the true and false ways of seeking the truth. He thereupon attempted to apply mathematical reasoning to all sciences. Comparing the mysteries of nature with the laws of mathematics, he dared to hope that the secrets of both could be unlocked with the same key". 18

17a. Ibid.——— Page 276
Although Descartes' philosophy has long since been superseded, his mathematics, especially his analytical geometry, will remain a valued possession for ages to come. Though his philosophy has been superseded, it offers a very good example of how one's mathematical studies can influence his philosophy.

In much the same manner are mathematics and philosophy closely connected in the systems of Kant, of Leibniz, of Spinoza, and of a host of others who were mathematicians and at the same time philosophers.

In writing this thesis I have shown the relations existing between mathematics and philosophy by the use of a three-fold scheme. First, I disclosed how philosophy makes of mathematics a true and a practical science. I indicated this by showing the absolute and necessary dependence of mathematics upon metaphysics. Secondly, I revealed the definite relations between mathematics and life. I gave illustrations of this relationship in ancient times and in modern times and showed how this relationship was necessary for ordered society. I showed the dire results of our getting away from this relationship at present. Thirdly, I proved how mathematics was in itself a philosophy and told how it justified its right of existance. I pointed out its attractiveness to thinkers on account of its self-sufficiency, its utility,
and its long and glorious history. Then I gave examples of men whose philosophic views were affected by their mathematical knowledge. This scheme gives an indication of the relations between mathematics and philosophy. One who wishes a more complete view of these relations and their significance should consult the bibliography at the end.

My one aim in the writing of this thesis has been to show this close and necessary relationship existing between mathematics and philosophy. I will be satisfied if I have given the reader some idea of this relationship.

Finis
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