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Exploring the Natural Connection of Mathematical and Musical Concepts Using Mathematica

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Exploring the Natural Connection of
Mathematical and Musical Concepts
Using Mathematica

Submitted in Partial Fulfillment of the Requirements for Graduation With Honors to
the Department of Mathematics, Engineering, and Computer Science at Carroll College,
Helena, Montana

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Thanks again to everybody who supported, helped, and encouraged us while we were working on our thesis.
Abstract

Math and music share one very common property; they are both universal languages of the world. Their scientific relationship represents another powerful tie. The harmonious, unique, and enjoyable tunes we hear are surrounded by scientific and mathematical concepts that make music possible. Exploring this natural connection between math and music allows for a more complete understanding of both these subjects. By analyzing the formulas used to create musical notes, examining how to apply matrices to music, and implementing an inter-disciplinary unit of math and music into the classroom, the reader of this thesis will discover the multitude of similarities that math and music share.
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Introduction

The job of a teacher goes far beyond teaching the subject matter. In addition to providing students with appropriate knowledge of the subject, a teacher must also find ways to make the materials interesting. One way to do this is to provide activities that motivate students to actively participate in class. This includes a number of options from hands-on projects that are done in class to units that involve detailed work outside of the classroom. These activities can provide a higher level of thinking while keeping the attention of the students. The downside to hands-on projects are that they require much effort on the part of the teacher and they usually cannot be done on a daily basis due to lack of time and materials allowed to the teacher. For this reason, other methods must be devised to keep the students engaged in the material and motivated to learn.

This thesis provides such an option for teachers. The following pages present numerous concepts that could be integrated into a high school math class. Creating an interdisciplinary program that combines the two similar subjects of math and music is a great way to provide the students with learning that is both original and creative. This method also permits the teacher to avoid planning special activities every day. By integrating musical concepts into a daily lesson plan, the teacher can give a regular lecture while providing the students with materials that are stimulating and interesting.
How Math and Music Intertwine

Dr. Larry Solomon in his doctorate paper, *Symmetry as a Compositional Determinant*, states that music is a form of art and it is in this art form that composers and musicians both express their artistic skills [1]. Behind this creative process, however, lie fundamental mathematics and physics concepts. Through examining how these concepts provide the foundation for music, the reader will see the various qualities of sound that make it interesting and useful to understand.

Basics of Sound

Music stripped of all rhythm and form is better defined as sound [2]. Sound is produced by repeating sound waves. Progressively adding different sounds at different times creates musical composition. These compositions can be put together using certain rules, causing the music to be labeled as a certain style [3]. These compositions can also be put together in such a way to express the feelings and thoughts of the composer [3]. This creative form of communication dates far back in time, as does the math and physics that produce the sounds that are created and heard.

Vibrating substances produce sound. Air pressure is constant on a dormant object, but when the object is struck (such as a string, a drum head, or a door), it begins to vibrate at a certain rate. This vibration causes a change in air pressure around the object. These changes in air pressure spread away from the vibrating object in waves of increased pressure (compression) and decreased pressure (rarefaction), which are called compression waves. The human ear picks up these changes of pressure and hears the
sound. Because of this, compression waves caused by a vibrating object are called sound waves. Figure I illustrates the rarefactions and compressions of a sound wave coming off of a guitar.

![Figure I](image)

Every vibrating object emits sound waves, constantly surrounding everything on this planet with compression waves. The reason why people are not always annoyed by all the sound waves reaching the ear, though, is due to the fact that our ears only detect certain sounds. Whether we hear a certain sound or not is determined by a mathematical concept. In the following section, the ways that math and physics affect all sound will be discussed. Understanding these mathematical connections will create opportunities to implement interdisciplinary lessons in a mathematics classroom.
Sound Waves

Sound is the movement of waves. These waves can be modeled and analyzed through Trigonometry. Using a trigonometric function (such as sine or cosine) any wave can be produced. In trigonometry, it is established that there are four main parts to a sound wave: wavelength, period, amplitude, and frequency. In this section it will be described how each of these parts affect a wave and how formulas were used in Mathematica to create notes. The wavelength of a sound wave is the distance between any two successive equivalent points on the wave. The period is the time required for one wavelength to pass a certain point. The amplitude of a sound wave is represented by the height of the wave. The frequency, or rate at which the wave goes through one complete wavelength, is also an important part of sound.

Sounds can be created in Mathematica in many different ways. We chose the Sine function to model sound because the function's initial position is zero and the function will continue to oscillate indefinitely. Trigonometric functions, however, produce a longitudinal wave. This creates a problem because sound comes from compression waves. To deal with this, the Play function in Mathematica was used. This command takes the value of the longitudinal wave and changes it into an output for the speakers [4]. The value of the trigonometric wave is the distance that the speaker should move away from its initial resting stare. This movement (vibration) of the speakers produces the sound. Together, the Sine and Play functions in Mathematica create the sounds needed. By altering these fundamental functions we were able to transform these sounds in an ordered system such that the composition of music was achieved.
Frequency, Wavelength, and Period

The wavelength, period and frequency of a sound wave are each independent aspects of a trigonometric wave, however, all three of these aspects interact when a wave is modeled in Mathematica. The wavelength deals only with distance, while the other two properties of waves deal with the time to make one wavelength [5]. Because Mathematica has a default speed (rate), in which it reads the values of the waves from a function, a change in wavelength will result in a direct change in period (time to complete one wavelength). Because of this constant reading rate in Mathematica this change in period will then affect the frequency of the note that is played with the Play function. In effect, changing the wavelength of a trigonometric function will change the frequency of a sound using the Play function in Mathematica.

The frequency of a sound wave directly affects the pitch of the sound that is produced. Objects that have a high frequency have a high pitch, while objects that have a lower frequency have a lower pitch. The frequency of a sound wave is measured in Hertz, which represents waves per second. All the sounds or vibration frequencies that the human ear can detect occur in the range of 20 to 20,000 Hertz. Vibrations above that range are termed ultrasonic and vibrations below that range are called subsonic. Using this information about frequency along with some research on the property of certain waves allowed the creation of musical notes in the Mathematica program. The pitch of the note A is 440 Hertz. To get a sine wave to model this frequency, the period of the wave must be changed. The equation $\sin(2 \cdot \pi \cdot t)$ constructs one complete Sine wave for each value of $t$. By multiplying $2 \cdot \pi \cdot t$ by 440, a wave with frequency of 440 cycles per
second is produced for each value of t. Putting this information all together, it was found that the note A could be produced with the *Mathematica* equation below:

\[ A_1 = \sin(440 \cdot 2 \cdot \pi \cdot t) \]  

(Eq. 1)

The properties of octaves have many interesting mathematical connections. Although this will be developed later, it is important to see how an octave can be produced as well as other notes in *Mathematica*. Doubling the frequency of a sound produces a note that is exactly an octave above the original note. To get the next octave from the note A1, decrease the period by using the following equation:

\[ A_2 = \sin(2 \cdot 440 \cdot 2 \cdot \pi \cdot t) \]  

(Eq. 2)

Other mathematical properties of notes are explored later in this chapter. These properties were implemented in *Mathematica* to compose music.

**Pythagorean Proportions**

Pythagoras was the first person to notice the property of octaves as he experimented with the pitches of sound that were produced from strings of varying lengths. Plucking strings of different length, he found the pitch of the sounds produced had a lot to do with the proportions of the string lengths. Especially pleasing to the ear were the pitches created by the lengths of strings with the ratios of 2:1, 3:2, and 4:3 compared to the original length [6]. Harmony was the name later given to this pleasing collaboration of sound from the two notes being played together [7]. Octaves, strings with length proportion 2:1 played together were especially harmonious. Plucking a string with a string twice the length of the original was found most pleasing to Pythagoras.
The proportions that Pythagoras found greatly affected the way that Western culture represents and plays music today. In Western culture, there are twelve notes from one octave to another. These twelve notes are now present because Pythagoras's finding with the 2:1, 3:2, and 4:3 proportions. The harmony that is associated with the proportion of 3:2 is called a fifth, while a 4:3 generates a fourth. Pythagoras tried to make a system of notes so that the octave, fourth, and fifth could be heard [6]. While experimenting with this system of notes, Pythagoras became the first person to investigate the vast field of temperament, which is still being studied today [8]. He was able to apply his experiments with the pitches from different strings to produce a system of notes that is known today as the Pythagorean temperament [8].

The Pythagorean temperament is a system of music that has twelve notes in each octave. Although Pythagoras dealt only with simple proportions in his experimentation, exploring his temperament introduces many substantial mathematical connections. It has already been stated that doubling the frequency of a note would construct a note one octave above the original. Since each of the octaves must be contained in every temperament, the period in all temperaments must be multiplied by a power of two. Pythagoras's appeal in the pitches produced by strings of length 3:2 (or 1.5 x the original) or 4:3 (1.333 x the original) is not so easily produced through mathematical formulas [9].

Producing the proportions of 3:2 and 4:3 becomes difficult to determine mathematically, given the fact that all the octaves must be present within the temperament. This problem arises from approximating 1.5 and 1.333 with a power of 2. To produce good sounding fourths and fifths the following mathematical equations are used:
2^{a/m} = 1.5 where \( a \) = the certain notes in octave and \( m \) = number of notes in the octave

\( 2^{b/m} = 1.3 \) where \( b \) = the certain notes in octave and \( m \) = number of notes in the octave

The first two columns on Figure II show the closest approximations to a fifth with a given number of notes. Four of the smallest numbers of notes in an octave that could possibly bring out a good sounding fifth are 5, 7, 10, and 12. Producing a good sounding fourth requires the same mathematical analysis. Again 5, 7, 10, and 12 are the numbers of notes needed in an octave to produce a good sounding fourth. The following table displays the different values of frequency that are produced with different numbers of notes in an octave. Notice that twelve notes have the lowest total difference in approximating the frequency of fourths and fifths.

<table>
<thead>
<tr>
<th>Number of Notes in Octave</th>
<th>Closest to Fifth (1.5)</th>
<th>Frequency Closest to Fourth Difference (1.333)</th>
<th>Frequency Difference</th>
<th>Total Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>&quot;2&quot; ( = 2.000 )</td>
<td>0.500</td>
<td>&quot;2&quot; ( = 2.000 )</td>
<td>0.667</td>
</tr>
<tr>
<td>2</td>
<td>( 2^{(1/2)} ) ( = 1.414 )</td>
<td>0.086</td>
<td>( 2^{(1/2)} ) ( = 1.414 )</td>
<td>0.081</td>
</tr>
<tr>
<td>3</td>
<td>( 2^{(2/3)} ) ( = 1.587 )</td>
<td>0.087</td>
<td>( 2^{(2/3)} ) ( = 1.587 )</td>
<td>0.073</td>
</tr>
<tr>
<td>4</td>
<td>( 2^{(4/4)} ) ( = 1.414 )</td>
<td>0.086</td>
<td>( 2^{(4/4)} ) ( = 1.414 )</td>
<td>0.081</td>
</tr>
<tr>
<td>5</td>
<td>( 2^{(3/5)} ) ( = 1.516 )</td>
<td>0.016</td>
<td>( 2^{(3/5)} ) ( = 1.516 )</td>
<td>0.014</td>
</tr>
<tr>
<td>6</td>
<td>( 2^{(6/6)} ) ( = 1.414 )</td>
<td>0.086</td>
<td>( 2^{(6/6)} ) ( = 1.414 )</td>
<td>0.073</td>
</tr>
<tr>
<td>7</td>
<td>( 2^{(4/7)} ) ( = 1.486 )</td>
<td>0.014</td>
<td>( 2^{(4/7)} ) ( = 1.486 )</td>
<td>0.013</td>
</tr>
<tr>
<td>8</td>
<td>( 2^{(5/8)} ) ( = 1.542 )</td>
<td>0.042</td>
<td>( 2^{(5/8)} ) ( = 1.542 )</td>
<td>0.036</td>
</tr>
<tr>
<td>9</td>
<td>( 2^{(5/6)} ) ( = 1.470 )</td>
<td>0.030</td>
<td>( 2^{(5/6)} ) ( = 1.470 )</td>
<td>0.027</td>
</tr>
<tr>
<td>10</td>
<td>( 2^{(6/10)} ) ( = 1.516 )</td>
<td>0.016</td>
<td>( 2^{(6/10)} ) ( = 1.516 )</td>
<td>0.014</td>
</tr>
<tr>
<td>11</td>
<td>( 2^{(7/11)} ) ( = 1.554 )</td>
<td>0.054</td>
<td>( 2^{(7/11)} ) ( = 1.554 )</td>
<td>0.037</td>
</tr>
<tr>
<td>12</td>
<td>( 2^{(7/12)} ) ( = 1.498 )</td>
<td>0.002</td>
<td>( 2^{(7/12)} ) ( = 1.498 )</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Figure II

Hence, twelve was chosen by later musician and mathematicians as the way to break up an octave. This way of breaking up an octave into twelve individual notes is now called equal or Pythagorean temperament. These twelve notes also make up the chromatic scale that is the prevalent base for Western music today.
Although Western culture uses an octave system with twelve notes, the Chinese base their music on a system of five notes. This system of music, called five-tone temperament, also has pleasing harmony. Indian music is based on a system of 7 notes in an octave and is called a just temperament. Both of these systems produce music with a very different style, but looking at Figure II, they contain frequencies that closely approximate the true fifth and fourth values. There are different reasons, however, for the fact that different cultures take up different systems. Much of these differences do not have to do with the mathematics behind musical frequencies, but rather with each culture's different philosophies and religious ideas [3].

Looking at Figure III, a guitar, which is a Western instrument, it can be seen that there are twelve notes in an octave. Doubling the frequency of a note on the guitar is accomplished by splitting the vibrating string in half. Looking at this guitar, the fret with the two dots splits the full length of the string exactly in half. If a string were struck, blocking the string's vibration on the fret with two dots would produce the next octave. There are twelve frets that are spaced between the endpoint and the middle fret. This number of frets directly correlates to the number of notes in the Western musical scale from one octave to the next.

Figure III
Chromatic Scale

Using *Mathematica*, we were able to create each note of the chromatic scale by multiplying the frequency by $2^{(1/12)}$. These twelve notes make up the chromatic scale. There are many mathematical concepts that are linked with this creation. Figure IV shows how twelve notes in the octave (or the chromatic scale) can be displayed. This Figure was created in *Mathematica*.

![Figure IV](image)

Using Figure IV and the properties of musical notes it is possible to model modular addition and subtraction also referred to as “clock arithmetic”. Musical notes are represented here with letter values, but they can also be given numerical values. The distance between two consecutive musical notes is called a musical step. Figure IV illustrates that by increasing an octave, you play the same named note that you started with. Starting at the note C and increasing twelve musical steps around the chromatic scale will end on the C again. This is an example of the symmetry in music. Each note
can be played up the scale and after the eleventh note is played, the same scale is played except at a higher octave.

Another application of the chromatic scale is the idea of the unit circle [10]. Since each of the most common angle measures is present on this circle, it would be easy to correlate every letter to each angle measure (A, D# = 0°, A = 180°) [10].

Also dealing with this chromatic circle of notes we can also begin to look at the mathematical connections in graphing and in creating musical compositions. By giving each note value a number (C=0, #C=1, D=2...) a way in which to represent these notes in composition is created [1]. By assigning each note to an ordered pair, in which the first number is the order that the note appears in the composition and the second number is the chromatic number of the note, a simple musical composition can be easily converted into a list of ordered pairs. The pair (3,1) pronounces that the third note in a composition to be played is C#.

Timbre

Another aspect of sound is timbre, which is the distinction between two different forms of sound. It is easy to determine that a certain note coming from a piano sounds quite different from that of a tuba. Timbre is affected by the way the compression wave changes over time and by the composition of the wave during the initial steady state [5]. All sounds are made up not only of the objects note frequency, but contain a multitude of other frequencies above and below that give the sound its character or timbre [5]. Working in Mathematica, waves can be produced with varying timbres by adding other sine waves with different frequencies to the initial equation. Equation 3 produces a
sound wave that has the same note value as Equation one, but this wave has a different timbre.

\[ \sin\left(440 \cdot 2 \cdot \pi \cdot 2^{12} \cdot t\right) + 3 \cdot \cos(555 \cdot \pi \cdot t) + 0.2 \cdot \tan(334 \cdot \pi \cdot t) \]  
(Eq. 3)

Amplitude

Sounds also have a specific amplitude, or volume. The amplitude of the waves that the vibrating object produces will determine the loudness or softness of the sound. The amplitude of a sound wave is measured in decibels. This decibel system is based on a logarithmic scale. Hence a sound at 20 decibels is ten times louder than a sound at 10 decibels. A sound at 120 decibels is 1,000,000,000,000 times more powerful than the smallest audible sound. Humans can withstand all sounds that lie under 130 decibels (after this it becomes painful) [11].

Rhythm

Rhythm in music provides the most direct correlation to math. All music produced has some sort of rhythm or it would just be sound. Music is most often produced in times of 2:4, 4:4, 3:4, 4:5. Each of these rhythms can be directly linked to a proportion or a fraction.

Overview

Adding in the general topics discussed above (sound waves, frequency, wavelength, period, amplitude, rhythm, and timbre) helps students identify the mathematical concepts that lie beneath the physics of making music. It was noted that
concepts such as mod addition and subtraction, fractions, trigonometric formulas (sine, cosine, and tangent), angles, and proportions could all be observed and studied using the theme of music. Using music to teach these concepts allows students to experiment with and analyze different models. Through this experimentation, reasoning skills become sharper as students investigate, explore, reflect, and make connections with the patterns created by combining musical and mathematical concepts. Craig Johnson, in his article, Functions of Number Theory in Music, states “The occurrence of patterns is the lifeblood of the permeation of mathematics through our existence, and nowhere are the creation and “feel” for patterns more prevalent than in the composition and enjoyment of music [12].”
Mathematical Symmetries

Any transformation performed on an object that results in the same figure as the original or its mirror image is known as a symmetry operation. In geometry, symmetry can be found by applying a translation, rotation, or reflection to an object. Each of the previously listed operations has a unique mathematical formula that can be applied to any geometrical figure to change its orientation.

Translation

A translation "slides" an object a fixed distance in a given direction. The original object and its translation have the same shape and size, and they face in the same direction. After an object is translated, its new position will be either to the left or right or above or below the original object. A translation is performed by applying the two formulas found below. The first one changes the value of $x$, moving the figure left or right across the horizontal axis. The second equation moves the figure up and down along the vertical axis by changing the $y$-value. Figure V shows an example of a simple mathematical transformation.

$$F(x, y) = (x + a, y)$$
$$F(x, y) = (x, y + a)$$

![Figure V](image-url)
Reflection

A reflection produces the mirror image of an object over a given line. The original object and its reflection are the same shape and size, but they face in opposite directions. Reflections are performed by applying the following formulas:

\[ T(x, y) = (-x, y) \]
\[ T(x, y) = (x, -y) \]

The first equation reflects an object over the vertical axis by changing the value of the \( x \). The second equation assigns a negative value to the \( y \), which reflects the object over the horizontal axis.

An example of a geometrical reflection is shown below in Figure VI.

![Diagram of reflections](image-url)

Figure VI.
Rotation

A rotation "spins" an object around a given point. This transformation keeps the image identical to the original, but "turns" it to a different angle. Any angle can be used when performing a rotation, but it is only useful to deal with the three most popular angles: 90°, 180°, and 270°. This operation is done by applying the following formulas:

For 90° rotation: \( F(x, y) = (y - x) \)
For 180° rotation: \( F(x, y) = (-x, -y) \)
For 270° rotation: \( F(x, y) = (-y, x) \)

Figure VII illustrates a clockwise rotation of 90° around the origin.

![Figure VII](image)

The past section summarizes some basic ideas about mathematical symmetries. It is very fundamental information and needs to be understood in order to look at what happens when these mathematical symmetry operations are performed on musical notes. The following section will explore the results of what happens when each of these formulas are applied to a musical note or a sequence of musical notes.
Musical Symmetry

It is possible to apply the three symmetrical operations: translations, reflections, and rotations, to a single note or a chain of notes. By using these mathematical operations, patterns can be created in the music that are pleasing to listen to.

How To Represent the Coordinates of a Musical Note

In order to apply these mathematical concepts, a musical note needs to be represented on a coordinate plane. On a normal coordinate plane, the numbers on the horizontal axis are characterized by the letter x. The letter y denotes the numbers on the vertical axis. Using these letters creates the coordinate \((x, y)\). In order for this theory to work, the notes must be assigned a similar value. When doing this, it is helpful to recall the chromatic scale chart that was previously discussed. (Figure IV).

When assigning a musical note values of \(x\) and \(y\), the first coordinate represents the order of the note in the melody. The second coordinate is the chromatic number of the note. Therefore, if the first note in the melody were a C, it would have the coordinates \((1,0)\). The one signifies that the C is the first note in the sequence, and the zero is assigned because that is the value of the chromatic number for a C note. Once the notes are represented as a coordinate pair, any of the following mathematical operations can be applied to the notes.
Translation in Music

A translation in music is used to repeat a melody or a single note later in a given song. A translation "slides" a note or a group of notes (a melody) a fixed distance in a given direction. The translated notes are the same as the original notes, but are found in a different location in the piece of music. Recall that the formula for a translation is:

\[ F(x,y) = F(x + a, y) \]

The following example shows what a translation in music looks like.

Figure VIII shows a sequence containing four notes. The first note in this sequence is an A. The second is a C, the next an E, and the final note is an A (one octave higher than the first). Assigning coordinates to these notes creates the ordered pair (1,9) for the first note in this string of notes. The one is assigned because it is the first note, and the nine because the note is an A, which is its chromatic number. The second note would have the coordinates (2,0), the E would become (3,4), and the octave A would be assigned the value of (4,9). Moving each of these notes four spaces to the right would give them a new coordinate by applying the translation formula to each coordinate. The new coordinates would be (5,9), (6,0), (7,4), and (8,9) for the notes A, C, E, and octave A, respectively. Doing this repeatedly would produce a melody with these four notes recurring over and over (Figure IX).
Reflection in Music

There are two types of reflections in music that create harmonic melodies and chords. They are tonal (horizontal reflection) and temporal (vertical reflection). A tonal reflection involves symmetry across a horizontal axis. This type of reflection preserves the “steps” between a note and the axis of reflection. A temporal reflection involves symmetry across the vertical axis, and is used to produce notes that are symmetrical with respect to time.

Tonal Reflection

A tonal reflection allows notes to be added to the original note to make a chord. Adding the appropriate notes while keeping the chord symmetrical with respect to the original note will produce a delightful sound. In order to create vertical symmetry, the steps between notes must be preserved. Once again, it is beneficial to take a look at the Chromatic Scale (Figure IV). When looking at a drawing of notes, they don’t always look symmetrical. This is because the note “steps” must be taken into account, not just the lines. This is due to the fact that two notes can share a line yet have a different value depending on if the note is sharp or flat. On a piano keyboard, it can be seen that there are half steps (represented by the black keys) between some of the white keys. When music is written, these “half steps” are taken into account by creating a scale that has eight notes in the octave. In this scale, it takes a half of a step to get to the next note. However, when we created our Chromatic Scale, a step size of one was used, giving each note its own value. The main reason for doing this was to make the numbers easier to work with.
The example below demonstrates a tonal reflection about the note A.

![Tonal Reflection Diagram]

**Figure X.**

As shown in Figure X, the A note was chosen to reflect the notes around. Therefore, each note about the A will have the same step size as the corresponding note below the A. Moving from the A to the C# takes a move of 4 "half-steps"; Going down to the F takes four "half-steps" as well. The same applies for each note in this series of notes. A jump of five "half-steps" is needed to reach the D above the A and the E below the A. To arrive at the B and the G, ten "half steps" are required up and down, respectively for each note. Since this chord was designed to be symmetrical with respect to the original note, it will have a harmonic, enjoyable sound.

Temporal Reflection

A temporal reflection involves symmetry across a vertical axis. This type of reflection produces notes that are symmetrical with respect to time. The phrase "with respect to time" refers to the notes being played individually, rather than in the previous example, where they were all played at the same time. When a temporal reflection is applied to a series of notes, the result is simply the playing of the melody backwards. When performing a temporal reflection, a note from the melody must be selected to be
the axis. As you can see in figure XI, the fourth note (C) in this sequence was selected to represent the y-axis. This note was then assigned the value of zero for its x-coordinate. Assigning these values allows this melody to be looked at as the x y plane. The notes before the C are given numbers according to their position on the plane. For example, the low C = -3, the F = -2, and the A = -1. Now each note is labeled as an ordered pair. The first coordinate represents the order of the note in the melody (according to the chosen axis that was), and the second coordinate symbolizes the chromatic number of the note. The notes now become coordinates with the following values: C = (-3, 0), F = (-2, 5), A = (-1, 9), and C = (0,0). The formula for a temporal reflection shows that:

\[ F(x, y) = F(-x, y) \]

Using this formula to reflect the notes over the axis, the outcome would be the values (1,9), (2,5), and (3,0), which are equivalent to the notes A, F, and C. Figures XI and XII illustrate what a temporal reflection looks like in music. When the notes are reflected over the imaginary axis drawn at C, the melody that is created is a mirror image of the first four notes, which simply plays the notes, and then plays them backward.
A rotation is the most complicated operation to perform on music, but is most frequently used by composers. An 180° rotation is the most useful rotation when looking at musical notes. It is also the most practical since most of the other rotations would position the notes awkwardly on the bars and at the wrong angles. A rotation of 180° is used to change the key of a song. In mathematical terms, this is equivalent to moving the note or series of notes up a given “step”. The number of notes in the melody determines the step size. Figure XIII shows an example of a rotation of 180° in music. In this case, the melody is eight notes in length. Therefore, rotating around the eighth note will produce a new melody that is eight steps higher (or an octave).

![Figure XIII](image)

Three different mathematical operations and how each is used to transpose musical notes have been observed in this section. Every one has its own unique property that can be used to compose an entertaining and satisfying piece of music. A translation repeats a note or tune later in the song, a reflection creates a symmetrical chord or plays a melody backward, and a rotation changes the key of a song by moving the notes up a decided amount of steps. Composers use these symmetrical operations every day, even if
they don’t realize that they are using these mathematical concepts to produce their music. Johan Sebastian Bach used symmetrical operations to create most of his musical pieces, and it seems that he was fully aware of the methods he was using. When he would sit down to write a musical piece, he would begin with eight original notes. These eight notes would give all the necessary pitch and rhythmic information he needed to generate an entire 8-voiced canon. He would then perform the operations of transposition, such as translations, reflections, and rotations, to create many of the masterpieces he is well known for today. Below is an example of one of Bach’s many compositions.

The original set of eight notes.

The 8-voiced canon produced using the original eight notes shown above.
Overview

This section provided a look at the different ways that symmetrical operations such as translations, rotations, and reflections can be used to transform musical notes and melodies into pieces of music that sounds enjoyable to the human ear. The concepts discussed could easily be put into a lesson plan for high school classroom. The first day would need to be spent understanding how each note can be written as an ordered pair. It would also be valuable for the students to become familiar with the Chromatic Scale and all of its properties. This concept is easily taught when dealing with modular arithmetic since it so closely related. Once this topic is mastered, the lesson plan would follow the same order as any normal geometry lesson plan on transformations. The change would occur only in the objects that are being transformed. Instead of looking at geometrical figures, the students would be dealing with musical notes. Each formula would be taught, and then the noted would be chosen, graphed, and transformed. A sample lesson plan on how to teach reflections can be found in Index 1.

At the end of an ordinary lesson on symmetrical operations, the students usually make use of what they have learned by doing some type of art project. Many teachers have students create a book cover or wallpaper using the methods of translations, rotations, and reflections. By applying these operations to a figure, they can create a beautiful and creative picture. However, if the students learn how to apply these symmetrical operations to music, at the end of the lesson they can create their own music. All that is needed is a keyboard. The notes can be written out on paper, then played on a keyboard by either the math teacher or a music teacher. They can be recorded and taken home for all to hear the outstanding song that was created in math class. This is a fresh,
new activity that is guaranteed to hold the attention of the students. If computers are available, the students can plug the notes into the Mathematica program or any other computer application that allows them to see and hear their creation. This is a great use of technology if there are available computers. As the world advances, it becomes more and more important for the students to learn how to properly use computers.

The next section provides a layout of the work that needs to be done in Mathematica in order to create musical notes and apply symmetrical operations to them.
Mathematica Code

The following pages explain the code created in Mathematica, showing exactly how these symmetrical operations can be applied to musical notes. The majority of the code was written to create the musical notes and the staff lines that can be seen in Figure XIV. To construct the appropriate notes, methods from the previous section were used, such as looking at the page as a coordinate plane. Changing the y coordinate allowed the note to move to a new position on the staff, therefore changing the pitch of the note. We chose this method so that we could change the note without difficulty if we wanted to write a new melody. The middle note was centered around (0,0) because we found it easier to rotate the object when this note was the center.

Figure XIV.

Both a translation and a rotation were performed on this set of notes by applying the translation and rotation formulas that were discussed in the Mathematical Symmetries section. In order to make this work in Mathematica, the formulas were put into matrix form. A third variable, z, was also added, with the sole purpose of being a place-holder in the matrix. This variable was always given the value of one. It was used to change the position properly, but then deleted before the notes were graphed. The equations below
show the transformation of each formula into a matrix. We have also included pictures and a small portion of the code below.

Translation:

\[
F(x, y) = (x + a, y) \Rightarrow \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x + a \\ y \\ 1 \end{pmatrix}
\]

\[
F(x, y) = (x, y + a) \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y + a \\ 1 \end{pmatrix}
\]

Rotation:

Matrix for rotation of \( \theta \)°:

\[
\begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]

The following code was used to translate and rotate the chain of notes:

Reduce [shape_] := Table [{shape [[i, 1]], shape [[i, 2]], {i, 1, Length [shape]}]);
Clear [\( \theta \), isom, moved, xtr, ytr]
Isom [\( \theta \), xtr_, ytr_] = {Cos [\( \theta \)], Sin [\( \theta \)], xtr}, {Sin [\( \theta \)], Cos [\( \theta \)], ytr}, {0,0,1}];
Moved [pts_, \( \theta \), xtr_, ytr_]: = Table [isom [\( \theta \), xtr, ytr].pts[[i]], {i,1, Length[pts]}]

The Reduce shape command makes a list of the \( x \) and \( y \) of each coordinate. Isom is a code of the matrix that changes the position of the notes. To use this matrix for a translation, we let \( yrt = 0 \) and \( \theta = 0 \), then let \( xtr \) take on values of zero through twenty-five. This change in \( x \) is what made the notes shift to the right twenty-five units. When
the rotation was performed, the same matrix was used, but with new values for $xtr$, $ytr$, and $\theta$. We let $xtr = 0$ while $\theta$ took on values of zero through $\pi$. By letting $\theta$ change values, the notes were rotated around the note at the origin. The Moved command applied each symmetry operation to the notes. Multiplying the Isom by each note’s coordinate did this.

Music was then added to the picture by using the notes we created in Mathematica. It became rather challenging to make the notes play at the same time they were appearing on the screen. The way we finally got the sound of the note and the picture of the note to coincide was to change the increment at which $\sigma$ and $xtr$ changed. By playing around with these values, the speed at which the notes moved across the screen or spun around the axis could be altered. Once the appropriate speed was found, we used the Show command to perform everything together. This command allowed the original notes and the newly transformed notes to be shown while the music played. Below are a few frames from our Mathematica code, which show the rotation and translation that take place. We have also included a disk with our code on it, allowing you to hear the music as well as seeing the notes.
Translation of Notes in *Mathematica*

Step 1: 

Step 2: 

Step 3: 

Step 4: 

Step 5: 

Step 6: 

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Rotation of Notes in *Mathematica*

Step 1:

![Step 1:](image)

Step 2:

![Step 2:](image)

Step 3:

![Step 3:](image)

Step 4:

![Step 4:](image)

Step 5:

![Step 5:](image)

Step 6:

![Step 6:](image)
Implementation of a Math and Music Interdisciplinary Lesson

Math and music are so intertwined it is hard to tell where one begins and the other ends. As we have shown, math and music have uncountable connections to each other. Applied mathematics plays a central role in all of modern science and advanced technology. The symbiotic relationship between mathematics and its areas of application is ever growing. Because of this, we feel that it is important to include musical concepts in math classes at all levels—middle school, high school, and even the elementary stage. Creating a interdisciplinary program that combines math and music allows students with different learning styles to see things in a new and innovative way. Musical applications of mathematics are enriching for those who are visual learners as well as those who are auditory learners. Such an interdisciplinary program would emphasize the practical value of fundamental math by relating skills to the basic elements of music. Charity Khan, the writer of Math and Music: Harmonious Connections, states “Music gives beauty and another dimension to mathematics by giving life and emotion to the numbers and patterns” [13]. Allowing such emotion to flow through the classroom would create an atmosphere where students could learn to their full potential. When dealing with music, some of the fears of math are eased and students are able to use their imagination and creativity rather than feel the need to solve problems with one correct answer.

Many children struggle with math or even claim that they don’t like math. This is partially due to the fact that the answers are so black and white. When adding, subtracting, multiplying, and dividing, the answer is either wrong or right. If a child gets the wrong answer, he or she tends to give up. By using musical concepts, there is a tendency to pull away from the numbers, in turn avoiding “right” or “wrong” answers.
Instead, the music that is created is observed and analyzed, which leads to a much more positive attitude toward the subject.

Gottfried Leibniz once said, “Music is the pleasure of the human soul experiences from counting without being aware that it is counting.” This is exactly what we need for students, especially for those who don’t enjoy math. If we can create an atmosphere where the students are learning the math skills they need without even realizing that they are doing math, they will not only have more fun, but might even become fond of math. There are also benefits to bringing music into the classroom.

The Mozart Effect

The Mozart Effect is a proposed theory that children have a significant increase in brain development when listening to the rhythmical music of Wolfgang Amadeus Mozart. This theory first came out of a study done in 1993 and has yet to be duplicated with similar results [12]. Recently there has been strong hesitation in trying to quantify an individual’s intelligence because it is hard to measure objectively [12]. Researchers have found little evidence that music learning automatically affects other cognitive skills. However, many studies since have produced significant results of music being tied to behavior [12].

Nonetheless, it has been established that the presence of music does trigger physical changes within the brain [12]. These changes, however, have not been shown in any direct way to induce more pleasant functioning of the brain (better spatial temporal reasoning) [12]. Scientists have found that music may be responsible for affecting mood. Today scientists are spending more time researching the effect that music has upon the
mood of the listener. Altering the mood of the learner may not only help students with their test taking abilities, but may also increase students' attention spans, and promote willingness to work with others [12].

Another belief in the benefits of music in the classroom is that all parts of the body vibrate at a certain frequency [12]. This position believes that the frequencies in classical music can alter the frequency of the body, thus bringing these frequencies into sync and resulting in a healthier and more alert individual. Music motivates, inspires, and de-stresses the mind. By using an interdisciplinary unit that combines mathematical and musical concepts, the physical needs of the brain are met while learning the appropriate math skills needed.
Conclusion

Choosing the right topic for our senior thesis was a complicated task. We are both in the field of education so we wanted to choose a topic that would be beneficial to us as high school math teachers. We both enjoy music as well, with Joe playing the guitar and Sarah playing the piano. When we ran into the topic of math and music, it intrigued us, and it sounded like an interesting theme to further pursue. When we started searching on the Internet and checking out books on math and music, we found an unbelievable amount of information. It was all so interesting and enthralling that we didn’t know where to stop. The more we looked, the more we found. We finally decided to focus on the geometry of transformations as applied to music. The decision for this was that it was interesting, yet not so complicated that we couldn’t teach the concepts to high school students. We have enjoyed the insights that we have gained while working with this topic and we look forward to implementing this information into the classroom when we become high school teachers ourselves.
Sources Cited


Index 1: A Sample Lesson Plan

Date: Monday, April 22, 2002
Subject: Algebra I
Topic: Symmetrical Operations
Lesson Name: Performing Reflections on Musical Notes
Unit Name: Mirror Images

Objectives:
- The students will identify a note by its proper chromatic number.
- The students will convert musical notes into an ordered pair of coordinates.
- The students will recognize the mirror image of a chain of musical notes.
- The students will draw the mirror image of musical notes.

Rationale:
- “Math” figures aren’t the only objects that can be reflected
- Many times it is useful to look at other objects and the purpose of using reflections
  +) A reflection can be used on notes to compose music, which we will see

Motivating Activities:
- Ask these questions:
  +) We have looked at the reflection of many geometrical objects. Do you think that geometric shapes are the only figures that can be reflected across an axis?
  -) NO. Any object can have a mirror reflection, including musical notes
  +) Today we will investigate what kind of sounds will be produced when a reflection is applied to a chain of notes.

Instructional Procedures:
- Learn the chromatic values of each note (HANDOUT #1)
  +) discuss properties of the chromatic scale
    -) how it was decided to chose 12 notes
    -) why they are represented on a unit circle

- Learn how to write each note as an ordered pair
  +) 1st coordinate represents the order of the note in a song
  +) 2nd coordinate represents the chromatic number of the note

- Practice putting notes into ordered pairs (HANDOUT #2—questions 1-4)

**Now let’s see what happens when we use the reflection formulas to change the notes**
- Review formulas for a reflection
  +) \( F(x, y) = (-x, y) \)
  +) \( F(x, y) = (x, -y) \)

- Use these formulas to perform reflection on notes (HANDOUT #2—questions 5-7)
  +) Questions:
    -) can you see the reflection?
    -) does it look the same as a reflection of a geometrical figure?
    -) what do you think this will sound like?
      -) will the reflection sound the same as the original?
      -) how will it be different?
  +) Let’s play it on the keyboard and hear how it sounds.
    -) now let them go back and see if they answered the previous questions correctly.

- Give them some new notes and let them perform reflection on them
  +) see if they can figure out what the reflection will sound like

Homework:
- Have each student write their own set of notes and perform a reflection on them
- When they come to class the next day, they can be played to hear the music they have created.

Evaluation Procedures:
- While talking in class,
  +) listen for students who aren’t understanding how to write notes as an ordered pair.
  +) listen for students who don’t see the reflection or know how it would sound

- When students turn in their notes,
  +) make sure they properly labeled the ordered pair of each note
  +) make sure they performed the reflection on each note correctly
  +) make sure they understand what their reflection would sound like

Closing Activities:
- Sample of how Bach uses this method to create music can be shown (HANDOUT #3)
- Put on some classical music and let the students listen and see if they can identify and places in the music where a reflection has occurred
  +) remind them that the sound of a reflection is just the notes being played, then played backwards
HANDOUT #1

THE CHROMATIC SCALE

Diagram of the chromatic scale with notes labeled from F# (0, -1) to C (0, 1), including other notes like G, A#, B, C#, D, D#, E, F, G#, A, and more.
Reflection in Music

1. Which note are we letting be the axis?

2. What value should we assign to this note?

3. What would the ordered pair for the first note be?

4. What would the ordered pair for the second note be?

5. What kind of a reflection is this (x-axis or y-axis)?

6. Which formula should we use to reflect the notes?

7. What would the values of each new coordinate be after the reflection?
Believe it or not, this is the entire 8-voiced canon. The notation gives all the necessary pitch and rhythmic information, but the operations of transposition, time and pitch intervals, and symmetry operations must be determined by the performers. Here is one possible solution:
Index 2: Chromatic Scale *Mathematica* Code

points = Table[Point[{Cos[-n*Pi / 6 + Pi / 2], Sin[-n*Pi / 6 + Pi / 2]}], {n, 12}];


labels = Table[Text[StyleForm[notes[[n]], FontSize -> 14, FontFamily -> "Times"], {1.25*Cos[-n*Pi / 6 + Pi / 2], 1.25*Sin[-n*Pi / 6 + Pi / 2]}, {0, 0.5}, {n, 12}];

figure1 = Show[
    Graphics[
        {{{PointSize[.04], points}, Circle[{0, 0}, 1], labels}}],
        PlotRange -> All, AspectRatio -> Automatic]

![Chromatic Scale Diagram]
Computer Generated Notes

Quarter notes

\[ A0 = \text{Play}[\sin(220 \cdot 2 \cdot \pi \cdot t), \{t, 0, 1\}] \]

\[ AS0 = \text{Play}[\sin(440 \cdot \pi \cdot 2^{(1/12)} \cdot t), \{t, 0, 1\}] \]

\[ B0 = \text{Play}[\sin(440 \cdot \pi \cdot 2^{(2/12)} \cdot t), \{t, 0, 1\}] \]
C1 = Play[Sin[440 Pi 2^ (3/12) t], {t, 0, 1}]

CS1 = Play[Sin[440 Pi 2^ (4/12) t], {t, 0, 1}]

D1 = Play[Sin[440 Pi 2^ (5/12) t], {t, 0, 1}]
DS1 = Play[Sin[440 Pi * 2^(6/12)*t], {t, 0, 1}]

E1 = Play[Sin[440 Pi * 2^(7/12)*t], {t, 0, 1}]

F1 = Play[Sin[440 Pi * 2^(8/12)*t], {t, 0, 1}]
FS1 = Play[Sin[440 * Pi * 2^(9/12) * t], {t, 0, 1}]

Gl = Play[Sin[440 * Pi * 2^(10/12) * t], {t, 0, 1}]

GS1 = Play[Sin[440 * Pi * 2^(11/12) * t], {t, 0, 1}]
A1 = Play[Sin[440 \cdot 2 \cdot \Pi \cdot t], \{t, 0, 1\}]

(The rest of the graphics of the play functions will be left out to conserve space.)

AS1 = Play[Sin[440 \cdot 2 \cdot \Pi \cdot t \cdot 2^{(1/12)}], \{t, 0, 1\}]

B1 = Play[Sin[440 \cdot 2 \cdot \Pi \cdot t \cdot 2^{(2/12)}], \{t, 0, 1\}]

C2 = Play[Sin[440 \cdot \Pi \cdot 2 \cdot t \cdot 2^{(3/12)}], \{t, 0, 1\}]

CS2 = Play[Sin[440 \cdot 2 \cdot \Pi \cdot t \cdot 2^{(4/12)}], \{t, 0, 1\}]

D2 = Play[Sin[440 \cdot 2 \cdot \Pi \cdot t \cdot 2^{(5/12)}], \{t, 0, 1\}]

DS2 = Play[Sin[440 \cdot 2 \cdot \Pi \cdot t \cdot 2^{(6/12)}], \{t, 0, 1\}]

E2 = Play[Sin[440 \cdot 2 \cdot \Pi \cdot t \cdot 2^{(7/12)}], \{t, 0, 1\}]

F2 = Play[Sin[440 \cdot 2 \cdot \Pi \cdot t \cdot 2^{(8/12)}], \{t, 0, 1\}]

FS2 = Play[Sin[440 \cdot 2 \cdot \Pi \cdot t \cdot 2^{(9/12)}], \{t, 0, 1\}]

G2 = Play[Sin[440 \cdot 2 \cdot \Pi \cdot t \cdot 2^{(10/12)}], \{t, 0, 1\}]

GS2 = Play[Sin[440 \cdot 2 \cdot \Pi \cdot t \cdot 2^{(11/12)}], \{t, 0, 1\}]

AS2 = Play[Sin[440 \cdot 4 \cdot \Pi \cdot t \cdot 2^{(1/12)}], \{t, 0, 1\}]

B2 = Play[Sin[440 \cdot 4 \cdot \Pi \cdot t \cdot 2^{(2/12)}], \{t, 0, 1\}]

C3 = Play[Sin[440 \cdot 4 \cdot \Pi \cdot t \cdot 2^{(3/12)}], \{t, 0, 1\}]

CS3 = Play[Sin[440 \cdot 4 \cdot \Pi \cdot t \cdot 2^{(4/12)}], \{t, 0, 1\}]

D3 = Play[Sin[440 \cdot 4 \cdot \Pi \cdot t \cdot 2^{(5/12)}], \{t, 0, 1\}]

G3 = Play[Sin[440 \cdot 4 \cdot \Pi \cdot t \cdot 2^{(6/12)}], \{t, 0, 1\}]

GS3 = Play[Sin[440 \cdot 4 \cdot \Pi \cdot t \cdot 2^{(7/12)}], \{t, 0, 1\}]

AS3 = Play[Sin[440 \cdot 4 \cdot \Pi \cdot t \cdot 2^{(8/12)}], \{t, 0, 1\}]

B3 = Play[Sin[440 \cdot 4 \cdot \Pi \cdot t \cdot 2^{(9/12)}], \{t, 0, 1\}]

C4 = Play[Sin[440 \cdot 4 \cdot \Pi \cdot t \cdot 2^{(10/12)}], \{t, 0, 1\}]

CS4 = Play[Sin[440 \cdot 4 \cdot \Pi \cdot t \cdot 2^{(11/12)}], \{t, 0, 1\}]

D4 = Play[Sin[440 \cdot 4 \cdot \Pi \cdot t \cdot 2^{(12/12)}], \{t, 0, 1\}]
DS3 = Play[\sin[440 \cdot 4 \cdot \pi \cdot t \cdot 2^\left(6/12\right)], \{t, 0, 1\}]

E3 = Play[\sin[440 \cdot 4 \cdot \pi \cdot t \cdot 2^\left(7/12\right)], \{t, 0, 1\}]

F3 = Play[\sin[440 \cdot 4 \cdot \pi \cdot t \cdot 2^\left(8/12\right)], \{t, 0, 1\}]

FS3 = Play[\sin[440 \cdot 4 \cdot \pi \cdot t \cdot 2^\left(9/12\right)], \{t, 0, 1\}]

G3 = Play[\sin[440 \cdot 4 \cdot \pi \cdot t \cdot 2^\left(10/12\right)], \{t, 0, 1\}]

GS3 = Play[\sin[440 \cdot 4 \cdot \pi \cdot t \cdot 2^\left(11/12\right)], \{t, 0, 1\}]

A3 = Play[\sin[440 \cdot 8 \cdot \pi \cdot t], \{t, 0, 1\}]

**Eighth notes**

TA0 = Play[\sin[440 \cdot \pi \cdot t], \{t, 0, 0.5\}]

TAS0 = Play[\sin[440 \cdot 2 \cdot \pi \cdot t \cdot 2^\left(1/12\right)], \{t, 0, 0.5\}]

TAS0 = Play[\sin[440 \cdot 2 \cdot \pi \cdot t \cdot 2^\left(2/12\right)], \{t, 0, 0.5\}]

TB0 = Play[\sin[440 \cdot 2 \cdot \pi \cdot t \cdot 2^\left(3/12\right)], \{t, 0, 0.5\}]

TC1 = Play[\sin[440 \cdot 2 \cdot \pi \cdot t \cdot 2^\left(4/12\right)], \{t, 0, 0.5\}]

TCS1 = Play[\sin[440 \cdot 2 \cdot \pi \cdot t \cdot 2^\left(5/12\right)], \{t, 0, 0.5\}]

TD1 = Play[\sin[440 \cdot 2 \cdot \pi \cdot t \cdot 2^\left(6/12\right)], \{t, 0, 0.5\}]

TDS1 = Play[\sin[440 \cdot 2 \cdot \pi \cdot t \cdot 2^\left(7/12\right)], \{t, 0, 0.5\}]

TE1 = Play[\sin[440 \cdot 2 \cdot \pi \cdot t \cdot 2^\left(8/12\right)], \{t, 0, 0.5\}]

TF1 = Play[\sin[440 \cdot 2 \cdot \pi \cdot t \cdot 2^\left(9/12\right)], \{t, 0, 0.5\}]

TFS1 = Play[\sin[440 \cdot 2 \cdot \pi \cdot t \cdot 2^\left(10/12\right)], \{t, 0, 0.5\}]

TG1 = Play[\sin[440 \cdot 2 \cdot \pi \cdot t \cdot 2^\left(11/12\right)], \{t, 0, 0.5\}]

TGS1 = Play[\sin[440 \cdot 2 \cdot \pi \cdot t \cdot 2^\left(12/12\right)], \{t, 0, 0.5\}]

TA1 = Play[\sin[440 \cdot 4 \cdot \pi \cdot t], \{t, 0, 0.5\}]

TAS1 = Play[\sin[440 \cdot 4 \cdot \pi \cdot t \cdot 2^\left(1/12\right)], \{t, 0, 0.5\}]

TB1 = Play[\sin[440 \cdot 4 \cdot \pi \cdot t \cdot 2^\left(2/12\right)], \{t, 0, 0.5\}]

TC2 = Play[\sin[440 \cdot 4 \cdot \pi \cdot t \cdot 2^\left(3/12\right)], \{t, 0, 0.5\}]

TCS2 = Play[\sin[440 \cdot 4 \cdot \pi \cdot t \cdot 2^\left(4/12\right)], \{t, 0, 0.5\}]

TD2 = Play[\sin[440 \cdot 4 \cdot \pi \cdot t \cdot 2^\left(5/12\right)], \{t, 0, 0.5\}]

TDS2 = Play[\sin[440 \cdot 4 \cdot \pi \cdot t \cdot 2^\left(6/12\right)], \{t, 0, 0.5\}]

TE2 = Play[\sin[440 \cdot 4 \cdot \pi \cdot t \cdot 2^\left(7/12\right)], \{t, 0, 0.5\}]
TF2=Play[Sin[440*4*t*Pi*2^8/(12)],{t,0,.5}]
TFS2=Play[Sin[440*4*t*Pi*2^9/(12)],{t,0,.5}]
TG2=Play[Sin[440*4*t*Pi*2^10/(12)],{t,0,.5}]
TGS2=Play[Sin[440*4*t*Pi*2^11/(12)],{t,0,.5}]
TA2=Play[Sin[440*8*t*Pi],{t,0,.5}]
TAS2=Play[Sin[440*8*t*Pi*2^1/(12)],{t,0,.5}]
TB2=Play[Sin[440*8*t*Pi*2^2/(12)],{t,0,.5}]
TC3=Play[Sin[440*8*t*Pi*2^3/(12)],{t,0,.5}]
TC3=Play[Sin[440*8*t*Pi*2^4/(12)],{t,0,.5}]
TD3=Play[Sin[440*8*t*Pi*2^5/(12)],{t,0,.5}]
TDS3=Play[Sin[440*8*t*Pi*2^6/(12)],{t,0,.5}]
TE3=Play[Sin[440*8*t*Pi*2^7/(12)],{t,0,.5}]
TF3=Play[Sin[440*8*t*Pi*2^8/(12)],{t,0,.5}]
TFS3=Play[Sin[440*8*t*Pi*2^9/(12)],{t,0,.5}]
TG3=Play[Sin[440*8*t*Pi*2^10/(12)],{t,0,.5}]
TGS3=Play[Sin[440*8*t*Pi*2^11/(12)],{t,0,.5}]
TA3=Play[Sin[440*8*t*Pi*16*t],{t,0,.5}]

Translations

Temporal

straight across = Show[B1, C2, G1, B1, C2, G1]
across and up3 = Show[B1, C2, G1, D2, DS2, AS1]

Tonal

Play[({Sin[Pi 2 440 t 2^2 (2/12)], Sin[t Pi 2 440 2^7 (7/12)], Sin[t Pi 2 440 2^8 (8/12)], Sin[t Pi 2 440 2^12 (12/12)], Sin[t Pi 2 440 2^16 (16/12)]}, {t, 0, 2}]

Play[({Sin[Pi 2 440 t 2^2 (2/12)], Sin[t Pi 2 440 2^7 (7/12)], Sin[t Pi 2 440 2^8 (8/12)], Sin[t Pi 2 440 2^12 (12/12)], Sin[t Pi 2 440 2^16 (16/12)], Sin[t Pi 2 440 2^20 (20/12)], Sin[t Pi 2 440 2^25 (25/12)], Sin[t Pi 2 440 2^30 (30/12)], Sin[t Pi 2 440 2^16 (16/12)]}, {t, 0, 2})]
Reflection

Show[B2, C3, G2, G2, C3, B2]

Song Reflection

saints = Show[TC2, TE2, TF2, TG2, TC2, TE2, TF2, TG2, TC2, TE2, TF2, TG2, TE2, TC2, TE2, TD2, 
TE2, TE2, TD2, TC2, TC2, TE2, TG2, TG2, TF2, TG2, TF2, TG2, TF2, TC2, TE2, TC2]

rsaints = Show[TF2, TF2, TG2, TG2, TG2, TE2, TC2, TC2, TD2, TE2, TD2, TE2, TC2, TE2, TG2, 
TF2, TE2, TC2, TG2, TF2, TE2, TC2, TG2, TF2, TE2, TC2]
Show[saints, rsaints]

Rotation

original = Show[B1, G1, D1, A1, C2]

rotation around C2 = Show[B1, G1, D1, A1, C2, DS2, A2, F2, CS2]
Song

\[ \text{space} = \text{Play}[2 \times, \{x, 0, .2\}] \]

\[ \text{we wish} = \text{Show}[\text{DS2, GS2, TGS1, TAS1, TG1, F2, F2, space, F2, RS2, TGS1, TC2, TAS1, T6S1, G2, DS2, space, DS2, C3, TC2, TC3, TC2, TAS1, GS2, F2, space, TDS1, TDS1, F2, RS2, G2, GS2}] \]

\[ \text{Show}[\text{TDS1, TDS1, F2, RS2, G2, GS2}] \]
Index 4: Translation Mathematica Code

\texttt{Off[General::spell]}
\texttt{Off[General::spell1]}
\texttt{do[x_] := Table[{x[[i, 1]], x[[i, 2]]}, {i, 1, Length[x]}]}
\texttt{undo[x_] := Table[{x[[i, 1]], x[[i, 2]]}, {i, 1, Length[x]}]}

\texttt{dline1pts = {{-10, 5, 1}, {20, 5, 1}}; line1pts = Table[{dline1pts[[i, 1]], dline1pts[[i, 2]]}, {i, 1, Length[dline1pts]}]; line1 = Line[line1pts];}
\texttt{dline2pts = {{-10, 1, 1}, {20, 1, 1}}; line2 = Table[{dline2pts[[i, 1]], dline2pts[[i, 2]]}, {i, 1, Length[dline2pts]}]; line2 = Line[line2pts];}
\texttt{dline3pts = {{-10, 1.5, 1}, {20, 1.5, 1}}; line3 = Table[{dline3pts[[i, 1]], dline3pts[[i, 2]]}, {i, 1, Length[dline3pts]}]; line3 = Line[line3pts];}
\texttt{dline4pts = {{-10, 2, 1}, {20, 2, 1}}; line4 = Table[{dline4pts[[i, 1]], dline4pts[[i, 2]]}, {i, 1, Length[dline4pts]}]; line4 = Line[line4pts];}
\texttt{dline5pts = {{-10, 2.5, 1}, {20, 2.5, 1}}; line5 = Table[{dline5pts[[i, 1]], dline5pts[[i, 2]]}, {i, 1, Length[dline5pts]}]; line5 = Line[line5pts];}
\texttt{dline6pts = {{-10, 3, 1}, {20, 3, 1}}; line6 = Table[{dline6pts[[i, 1]], dline6pts[[i, 2]]}, {i, 1, Length[dline6pts]}]; line6 = Line[line6pts];}
\texttt{dline7pts = {{-10, 3.5, 1}, {20, 3.5, 1}}; line7 = Table[{dline7pts[[i, 1]], dline7pts[[i, 2]]}, {i, 1, Length[dline7pts]}]; line7 = Line[line7pts];}
\texttt{dline8pts = {{-10, 4, 1}, {20, 4, 1}}; line8 = Table[{dline8pts[[i, 1]], dline8pts[[i, 2]]}, {i, 1, Length[dline8pts]}]; line8 = Line[line8pts];}
\texttt{dCnotepts = Table[.4*Cos[\theta] - 10, .2*Sin[\theta] + .5, 1, \theta, 0, 2*Pi, Pi/15]/N; Cnotepts = Table[dCnotepts[[i, 1]], dCnotepts[[i, 2]]], {i, 1, Length[dCnotepts]}]; Cnote = Line[dCnotepts];}
\texttt{dDnotepts = Table[.4*Cos[\theta] - 7, .2*Sin[\theta] + .75, 1, \theta, 0, 2*Pi, Pi/15]/N; Dnotepts = Table[dDnotepts[[i, 1]], dDnotepts[[i, 2]]], {i, 1, Length[dDnotepts]}]; Dnote = Line[dDnotepts];}
\texttt{dEnotepts = Table[.4*Cos[\theta] + 2, .2*Sin[\theta] + 1.5, 1, \theta, 0, 2*Pi, Pi/15]/N; Enotepts = Table[dEnotepts[[i, 1]], dEnotepts[[i, 2]]], {i, 1, Length[dEnotepts]}]; Enote = Line[dEnotepts];}
\texttt{dFnotepts = Table[.4*Cos[\theta] - 1, .2*Sin[\theta] + 1.25, 1, \theta, 0, 2*Pi, Pi/15]/N; Fnotepts = Table[dFnotepts[[i, 1]], dFnotepts[[i, 2]]], {i, 1, Length[dFnotepts]}]; Fnote = Line[dFnotepts];}
\texttt{dGnotepts = Table[.4*Cos[\theta] + 2, .2*Sin[\theta] + 1.5, 1, \theta, 0, 2*Pi, Pi/15]/N; Gnotepts = Table[dGnotepts[[i, 1]], dGnotepts[[i, 2]]], {i, 1, Length[dGnotepts]}]; Gnote = Line[dGnotepts];
When we created this set of notes, we didn’t worry about where they were centered. We started on the left-hand side with the note C, then went up and over until we had all of the notes of an octave. Although we created eight notes, we found that it worked better to only use the first three in our translation. In our next isometry, we changed the original notes to make it easier to rotate.
reduce[shape_] := Table[shape[[i, 1]], shape[[i, 2]]], {i, 1, Length[shape]]]; Clear[theta, isom, moved, xtr, ytr, theta]; isom[theta_, xtr_, ytr_] := {{Cos[theta], -Sin[theta], xtr}, {Sin[theta], Cos[theta], ytr}, {0, 0, 1}}; moved[pts_, theta_, xtr_, ytr_] := Table[isom[theta, xtr, ytr].pts[[LeftDoubleBracket]i[[RightDoubleBracket]], {i, 1, Length[pts]}]; shapelist = {}; ytr = 0; theta = 0; Do[ Clear[dnewpts, newpts]; dnewpts = moved[dCnotepts, theta, xtr, ytr]; newpts = Table[{dnewpts[[i, 1]], dnewpts[[i, 2]]}, {i, 1, Length[dnewpts]}]; newCnote = Line[newpts]; Clear[dnewpts, newpts]; dnewpts = moved[dDnotepts, theta, xtr, ytr]; newpts = Table[{dnewpts[[i, 1]], dnewpts[[i, 2]]}, {i, 1, Length[dnewpts]}]; newDnote = Line[newpts]; Clear[dnewpts, newpts]; dnewpts = moved[dEnotepts, theta, xtr, ytr]; newpts = Table[{dnewpts[[i, 1]], dnewpts[[i, 2]]}, {i, 1, Length[dnewpts]}]; newEnote = Line[newpts]; Clear[dnewpts, newpts]; dnewpts = moved[dFnotepts, theta, xtr, ytr]; newpts = Table[{dnewpts[[i, 1]], dnewpts[[i, 2]]}, {i, 1, Length[dnewpts]}]; newFnote = Line[newpts]; Clear[dnewpts, newpts]; dnewpts = moved[dGnotepts, theta, xtr, ytr]; newpts = Table[{dnewpts[[i, 1]], dnewpts[[i, 2]]}, {i, 1, Length[dnewpts]}]; newGnote = Line[newpts]; Clear[dnewpts, newpts]; dnewpts = moved[dAnotepts, theta, xtr, ytr]; newpts = Table[{dnewpts[[i, 1]], dnewpts[[i, 2]]}, {i, 1, Length[dnewpts]}]; newAnote = Line[newpts]; Clear[dnewpts, newpts]; dnewpts = moved[dBnotepts, theta, xtr, ytr]; newpts = Table[{dnewpts[[i, 1]], dnewpts[[i, 2]]}, {i, 1, Length[dnewpts]}]; newBnote = Line[newpts]; Clear[dnewpts, newpts]; dnewpts = moved[dHCnotepts, theta, xtr, ytr]; newpts = Table[{dnewpts[[i, 1]], dnewpts[[i, 2]]}, {i, 1, Length[dnewpts]}]; newHCnote = Line[newpts]; Clear[dnewpts, newpts];
\begin{verbatim}
dnewpts = moved[dCstempts, \theta, xtr, ytr];
newpts = Table[{dnewpts[[i, 1]], dnewpts[[i, 2]]}, {i, 1, Length[dnewpts]}];
newCstem = Line[newpts];
Clear[dnewpts, newpts];
dnewpts = moved[dDstempts, \theta, xtr, ytr];
newpts = Table[{dnewpts[[i, 1]], dnewpts[[i, 2]]}, {i, 1, Length[dnewpts]}];
newDstem = Line[newpts];
Clear[dnewpts, newpts];
dnewpts = moved[dEstempts, \theta, xtr, ytr];
newpts = Table[{dnewpts[[i, 1]], dnewpts[[i, 2]]}, {i, 1, Length[dnewpts]}];
newEstem = Line[newpts];
Clear[dnewpts, newpts];
dnewpts = moved[dFstempts, \theta, xtr, ytr];
newpts = Table[{dnewpts[[i, 1]], dnewpts[[i, 2]]}, {i, 1, Length[dnewpts]}];
newFstem = Line[newpts];
Clear[dnewpts, newpts];
dnewpts = moved[dGstempts, \theta, xtr, ytr];
newpts = Table[{dnewpts[[i, 1]], dnewpts[[i, 2]]}, {i, 1, Length[dnewpts]}];
newGstem = Line[newpts];
Clear[dnewpts, newpts];
dnewpts = moved[dAstempts, \theta, xtr, ytr];
newpts = Table[{dnewptsf[[i, 1]], dnewpts[[i, 2]]}, {i, 1, Length[dnewpts]}];
newAstem = Line[newpts];
Clear[dnewpts, newpts];
dnewpts = moved[dBstempts, \theta, xtr, ytr];
newpts = Table[{dnewpts[[i, 1]], dnewpts[[i, 2]]}, {i, 1, Length[dnewpts]}];
newBstem = Line[newpts];
Clear[dnewpts, newpts];
dnewpts = moved[dHCstempts, \theta, xtr, ytr];
newpts = Table[{dnewpts[[i, 1]], dnewpts[[i, 2]]}, {i, 1, Length[dnewpts]}];
newHCstem = Line[newpts];
newmusic = {linel, line2, line3, line4, line5, newCnote, newDnote, Cnote, Dnote,
             Enote, Cstem, Dstem, Estem, Enote, newCstem, newDstem, newEstem};
AppendTo[shapelist, newmusic],
{xtr, 0, 10, .51}]
\end{verbatim}

We moved these notes by applying the transformation matrix. This matrix focused on moving the x-coordinates, defined in our code by xtr in the Isom command. To show the transformation that took place, we inserted the original notes into the animation along with the moved notes. This shows how a translation effects a piece of music—by moving to a further place in the song.

p = Show[E1, F1, G1, E1, F1, G1]
shapelist = Show[{p,
    Table[Show[Graphics[shapelist[[i]]], AxesOrigin -> {-2, 0},
            PlotRange -> {{-11, 15}, {0, 4}}, AspectRatio -> Rule[5], {i, 1,
            Length[shapelist]}]}];
Index 5: Rotation Mathematica Code

Off[General::spell]
Off[General::spell1]
do[x_,l]:=Table[{x[[i,1]],x[[i,2]],1},{i,1,Length[x]}]
undo[x_,l]:=Table[{x[[i,1]],x[[i,2]]},{i,l,Length[x]}]

We found that the undo command was not needed when we graphed our picture. This could be due
to the fact that all of our z-coordinates are 1 so the computer ignored this dimension.

dline1pts={{-10,5,1},{20,5,1}};
line1pts=Table[{dline1pts[[i,1]],dline1pts[[i,2]]},{i,1,Length[dline1pts]}];
line1=Line[line1pts];
dline2pts={{-10,1,1},{20,1,1}};
line2pts=Table[{dline2pts[[i,1]],dline2pts[[i,2]]},{i,1,Length[dline2pts]}];
line2=Line[line2pts];
dline3pts={{-10,1.5,1},{20,1.5,1}};
line3pts=Table[{dline3pts[[i,1]],dline3pts[[i,2]]},{i,1,Length[dline3pts]}];
line3=Line[line3pts];
dline4pts={{-10,2,1},{20,2,1}};
line4pts=Table[{dline4pts[[i,1]],dline4pts[[i,2]]},{i,1,Length[dline4pts]}];
line4=Line[line4pts];
dline5pts={{-10,2.5,1},{20,2.5,1}};
line5pts=Table[{dline5pts[[i,1]],dline5pts[[i,2]]},{i,1,Length[dline5pts]}];
line5=Line[line5pts];
dline6pts={{-10,0.1},{20,0.1}};
line6pts=Table[{dline6pts[[i,1]],dline6pts[[i,2]]},{i,1,Length[dline6pts]}];
line6=Line[line6pts];
dline7pts={{-10,-.5,1},{20,-.5,1}};
line7pts=Table[{dline7pts[[i,1]],dline7pts[[i,2]]},{i,1,Length[dline7pts]}];
line7=Line[line7pts];
dline8pts={{-10,-1.1},{20,-1.1}};
line8pts=Table[{dline8pts[[i,1]],dline8pts[[i,2]]},{i,1,Length[dline8pts]}];
line8=Line[line8pts];
dline9pts={{-10,-1.5,1},{20,-1.5,1}};
line9pts=Table[{dline9pts[[i,1]],dline9pts[[i,2]]},{i,1,Length[dline9pts]}];
line9=Line[line9pts];
dCnotepts=Table[{.4*Cos[\[Theta]]-6,.2*Sin[\[Theta]]-.75,l},{\[Theta],0,2*Pi,Pi/15}]//N;
Cnotepts=Table[{dCnotepts[[i,1]],dCnotepts[[i,2]]},{i,1,Length[dCnotepts]}];
Cnote=Line[Cnotepts];
dDnotepts=Table[{.4*Cos[\[Theta]]-4,.2*Sin[\[Theta]]-.5,l},{\[Theta],0,2*Pi,Pi/15}]//N;
Dnotepts=Table[{dDnotepts[[i,1]],dDnotepts[[i,2]]},{i,1,Length[dDnotepts]}];
Dnote=Line[Dnotepts];
dEnotepts=Table[{.4*Cos[\[Theta]]-2,.2*Sin[\[Theta]]-.25,l},{\[Theta],0,2*Pi,Pi/15}]//N;
Enotepts=Table[{dEnotepts[[i,1]],dEnotepts[[i,2]]},{i,1,Length[dEnotepts]}];
Enote=Line[Enotepts];
dFnotepts=Table[{.4*Cos[\[Theta]],.2*Sin[\[Theta]],l},{\[Theta],0,2*Pi,Pi/15}]//N;
Fnotepts=Table[{dFnotepts[[i,1]],dFnotepts[[i,2]]},{i,1,Length[dFnotepts]}];
Fnote=Line[Fnotepts];
dGnotepts=
    Table[{
        .4*Cos[\[Theta]] + 2, .2*Sin[\[Theta]] + .25, 1, \[Theta] 
    }, \[Theta], 0, 2*Pi, Pi/15]/N;
Gnotepts=Table[{
    dGnotepts[[i, 1]], dGnotepts[[i, 2]]}, {i, 1, Length[dGnotepts]}];
Gnote=Line[Gnotepts];
dAnotepts=
    Table[{
        .4*Cos[\[Theta]] + 4, .2*Sin[\[Theta]] + .5, 1, \[Theta] 
    }, \[Theta], 0, 2*Pi, Pi/15]/N;
Anotepts=Table[{
    dAnotepts[[i, 1]], dAnotepts[[i, 2]]}, {i, 1, Length[dAnotepts]}];
Anote=Line[Anotepts];
dBnotepts=
    Table[{
        .4*Cos[\[Theta]] + 6, .2*Sin[\[Theta]] + .75, 1, \[Theta] 
    }, \[Theta], 0, 2*Pi, Pi/15]/N;
Bnotepts=Table[{
    dBnotepts[[i, 1]], dBnotepts[[i, 2]]}, {i, 1, Length[dBnotepts]}];
Bnote=Line[Bnotepts];
dHCnotepts=
    Table[{
        .4*Cos[\[Theta]] + 8, .2*Sin[\[Theta]] + 1, \[Theta] 
    }, \[Theta], 0, 2*Pi, Pi/15]/N;
HCnotepts=Table[{
    dHCnotepts[[i, 1]], dHCnotepts[[i, 2]]}, {i, 1, Length[dHCnotepts]}];
HCnote=Line[HCnotepts];
dCstempts={{-5.6, -.75, 1}, {-5.6, .25, 1}};
Cstempts=Table[{
    dCstempts[[i, 1]], dCstempts[[i, 2]]}, {i, 1, Length[dCstempts]}];
Cstem=Line[Cstempts];
dDstempts={{-3.6, -.5, 1}, {-3.6, .5, 1}};
Dstempts=Table[{
    dDstempts[[i, 1]], dDstempts[[i, 2]]}, {i, 1, Length[dDstempts]}];
Dstem=Line[Dstempts];
dEstempts={{-1.6, -.25, 1}, {-1.6, .75, 1}};
Estempts=Table[{
    dEstempts[[i, 1]], dEstempts[[i, 2]]}, {i, 1, Length[dEstempts]}];
Estem=Line[Estempts];
dFstempts={{.4, 0, 1}, {.4, 1, 1}};
Fstempts=Table[{
    dFstempts[[i, 1]], dFstempts[[i, 2]]}, {i, 1, Length[dFstempts]}];
Fstem=Line[Fstempts];
dGstempts={{2.4, .25, 1}, {2.4, 1.25, 1}};
Gstempts=Table[{
    dGstempts[[i, 1]], dGstempts[[i, 2]]}, {i, 1, Length[dGstempts]}];
Gstem=Line[Gstempts];
dAstempts={{4.4, .5, 1}, {4.4, 1.5, 1}};
Astempts=Table[{
    dAstempts[[i, 1]], dAstempts[[i, 2]]}, {i, 1, Length[dAstempts]}];
Astem=Line[Astempts];
dBstempts={{6.4, .75, 1}, {6.4, 1.75, 1}};
Bstempts=Table[{
    dBstempts[[i, 1]], dBstempts[[i, 2]]}, {i, 1, Length[dBstempts]}];
Bstem=Line[Bstempts];
dHCstempts={{8.4, 1, 1}, {8.4, 2, 1}};
HCstempts=Table[{
    dHCstempts[[i, 1]], dHCstempts[[i, 2]]}, {i, 1, Length[dHCstempts]}];
HCstem=Line[HCstempts];
music={linel, line2, line3, line4, line5, line6, line7, line8, line9, Cnote, Dnote, 
    Enote, Fnote, Gnote, Anote, Bnote, HCnote, Cstem, Dstem, Estem, Fstem, Gstem, 
    Astem, Bstem, HCstem};
song=Show[Graphics[music], AxesOrigin[{0, 0}], 
    PlotRange[{[-11, 12], [-2, 5]}, AspectRatio[Rule]0.5];
This code creates the musical notes and the staff lines. By changing the y-coordinates, we can make the note move to a different line, therefore changing the pitch of the note. This will come in handy when we want to write a song or little melody because we won’t have to change much of the code, just adjust the y-value. We centered the middle note around (0,0) because we found it easier to rotate the object when this note was in the center. When we first designed the notes, we started at the left-hand side but found that this made it difficult to rotate the object in our rotation matrix. We also found that it looked better when we increased the range and domain of the graph. This allowed more of the picture to be seen when we rotated the object.

The moved command applied the rotation matrix, defined as Isom, to our notes. This matrix rotated our notes 180 degrees counter-clockwise and flipped the note stems so they are facing down. This is interesting because when a piece of music is written, the note stems are drawn facing down when the note is above the C position. We also noticed that rotating around the 3rd note moves the piece of music up 3 steps, which is exactly what we talked about in our presentation—a rotation around the xth note of a single-step scale will move the notes up x number of steps.
dnewpts = moved[dGnotepts, theta, xtr, ytr];
newpts = Table[{dnewpts[[i,1]], dnewpts[[i,2]]}, {i,1,Length[dnewpts]}];
newGnote = Line[newpts];
Clear[dnewpts,newpts];
dnewpts = moved[dAnotepts, theta, xtr, ytr];
newpts = Table[{dnewpts[[i,1]], dnewpts[[i,2]]}, {i,1,Length[dnewpts]}];
newAnote = Line[newpts];
Clear[dnewpts,newpts];
dnewpts = moved[dBnotepts, theta, xtr, ytr];
newpts = Table[{dnewpts[[i,1]], dnewpts[[i,2]]}, {i,1,Length[dnewpts]}];
newBnote = Line[newpts];
Clear[dnewpts,newpts];
dnewpts = moved[dHCnotepts, theta, xtr, ytr];
newpts = Table[{dnewpts[[i,1]], dnewpts[[i,2]]}, {i,1,Length[dnewpts]}];
newHCnote = Line[newpts];
Clear[dnewpts,newpts];
dnewpts = moved[dCstempts, theta, xtr, ytr];
newpts = Table[{dnewpts[[i,1]], dnewpts[[i,2]]}, {i,1,Length[dnewpts]}];
newCstem = Line[newpts];
Clear[dnewpts,newpts];
dnewpts = moved[dDstempts, theta, xtr, ytr];
newpts = Table[{dnewpts[[i,1]], dnewpts[[i,2]]}, {i,1,Length[dnewpts]}];
newDstem = Line[newpts];
Clear[dnewpts,newpts];
dnewpts = moved[dEstempts, theta, xtr, ytr];
newpts = Table[{dnewpts[[i,1]], dnewpts[[i,2]]}, {i,1,Length[dnewpts]}];
newEstem = Line[newpts];
Clear[dnewpts,newpts];
dnewpts = moved[dFstempts, theta, xtr, ytr];
newpts = Table[{dnewpts[[i,1]], dnewpts[[i,2]]}, {i,1,Length[dnewpts]}];
newFstem = Line[newpts];
Clear[dnewpts,newpts];
dnewpts = moved[dGstempts, theta, xtr, ytr];
newpts = Table[{dnewpts[[i,1]], dnewpts[[i,2]]}, {i,1,Length[dnewpts]}];
newGstem = Line[newpts];
Clear[dnewpts,newpts];
dnewpts = moved[dAstempts, theta, xtr, ytr];
newpts = Table[{dnewpts[[i,1]], dnewpts[[i,2]]}, {i,1,Length[dnewpts]}];
newAstem = Line[newpts];
Clear[dnewpts,newpts];
dnewpts = moved[dBstempts, theta, xtr, ytr];
newpts = Table[{dnewpts[[i,1]], dnewpts[[i,2]]}, {i,1,Length[dnewpts]}];
newBstem = Line[newpts];
Clear[dnewpts,newpts];
newmusic = {line1, line2, line3, line4, line5, line6, line7, line8, line9, newCnote, 
            newDnote, newEnote, newCstem, newDstem, newEstem, Cnote, Dnote, Enote, Cstem, 
            Dstem, Estem}; AppendTo[shapelist,newmusic],{theta,0, Pi, Pi/22}]

We had to change the increment of the rotation to decrease the distance that the notes moved each frame in order to time the sound with the visual animation of the notes. We found that pi/22 works the best to synchronize our animation with the music. When each frame is done showing, the final frame shows each note's move.
Note = Show[B1, C2, CS2, D2, DS2, E2]

shapes = Show[Table[
    Show[Graphics[shapeList[[i]]], AxesOrigin -> {-2, 0},
    PlotRange -> {{-11, 15}, {-5, 4}}, AspectRatio -> 1],
    {i, 1, Length[shapeList]}, Note];}