Spring 2004

It’s Your Move: Optimal Playing Strategies for Mancala

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It's Your Move: Optimal Playing Strategies for Mancala

Kelly Beffert

Submitted in Partial Fulfillment of the Requirements for Graduation with Honors to the Department of Mathematics, Engineering, and Computer Science, Carroll College, Helena, Montana.

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April 5, 2004
Acknowledgments

I would like to extend my sincere thanks to my thesis director Dr. Mark Parker. His guidance during the research and writing process was invaluable and is greatly appreciated. I would also like to thank my readers, Dr. Jeff Morris and Dr. Holly Zullo for their helpful suggestions concerning the clarity and format of the document. I must thank my brother, Jason, for being my opponent, enduring countless hours of play to aid in my comprehension and knowledge of the game. Finally, my parents deserve the greatest thanks of all for providing their undying support in everything I do, I love you.
Abstract

This thesis explores topics in game theory within the framework of the popular game Mancala. Game theory is a branch of mathematical analysis developed to study decision making in conflict situations such as games. Mancala is believed to be one of the oldest games in the world, and makes a great study because of its many levels of complexity. I explore several variations of the game and demonstrate the increasing level of difficulty in developing optimal playing strategies.
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Chapter 1: Introduction

Introduction to Game Theory

Game theory is a branch of mathematics developed to study decision making in conflict situations.¹ These types of conflict situations exist when two or more people that have different or similar objectives act on the same system. For example, chess, bridge, and poker are games encompassed in game theory. The intriguing aspect of game theory, however, is that it abstracts from and generalizes the study of these types of traditional games, becoming applicable in a wide array of social situations such as problems of monopoly or economic warfare.²

John von Neumann is credited as the founder of game theory due to papers he published in 1928 and 1937.³ Recently, the mathematician Borel has been recognized for his contributions to the study in the early 1920's.³

Before proceeding with an analysis of a specific example, it is important to establish the definition of a game. As stated by Straffin, a game is any situation in which

i) There are at least two players. A player may be an individual, but it may also be a more general entity like a company, a nation, or a biological species.

ii) Each player has a number of possible strategies or courses of action which he or she may choose to follow.

iii) The strategies chosen by each player determine the outcome of the game.
iv) Associated with each possible outcome of the game is a collection of numerical payoffs, one to each player. These payoffs represent the value of the outcome to the different players.

Game theory provides a mathematical process for a player to select an optimal strategy when playing a game. In order for the theory to be applicable, a few ambiguities have been defined. First, game theory is the study of how players should rationally play games. Thus, the aim of each player is to end the game with an outcome that is the most favorable for him or her, meaning the outcome with the largest payoff. A player does have limited control over the outcome of the game because the player chooses which strategy he or she will employ. The final outcome is determined by the choices or strategies of all players involved. This is where the conflict of game theory enters. Therefore, rational play involves individual decisions regarding how to choose a strategy that produces a favorable outcome for you, realizing that other players are choosing strategies that will result in a favorable outcome for them.

Games analyzed by game theory are often divided into three classes: two-person zero-sum games, two-person non-zero-sum games, and n-person games. A two-person zero sum game occurs when each player knows his and his opponent's alternatives, the preference of each player is known, and sum of all payoffs are zero, as in two person poker. A two-person non-zero sum game is similar to a two-person zero sum game, the difference being the final payoff is not zero, as in the famous Prisoner's Dilemma. N-person games occur when there are more than two sets of conflicting interests, such as wide scale warfare. This thesis studies the optimal playing strategy for the two-person
zero-sum game Mancala. Mancala belongs to a category of games in which players make choices sequentially. In this division of games an optimal playing strategy can be modeled with a game tree.

A game tree, according to Straffin, is a mathematical model in which

i) Each node is labeled with the player who makes a choice at that node.

ii) Each branch leading downward from a node represents a possible choice made by the player at that node.

iii) Each branch corresponding to a choice made by a player is labeled with the probability that a player will make that choice.

iv) Each final node is labeled with payoffs to the players.

The utilization of game trees to model mathematical situations is not always ideal because game trees grow quite large very quickly. For example, in a 48-piece Mancala game, there are $6^4$ or 1,296 feasible board arrangements when Player 1’s third turn arises! This example highlights the combinatorial explosion that is typical of many game trees. In the analysis that follows, a game tree is used for an initial investigation, but the method is soon discarded in favor of actual play to study the tendencies and patterns inherent in Mancala.

**Introduction to Mancala**

Mancala is believed to be the oldest game in the world by many historians. The word Mancala means “to transfer” in Arabic. In playing Mancala, that is exactly what
you do; you transfer, or move, playing pieces from one pit to another. Mancala is coined "the African game of counting and strategy."¹ Some version of Mancala is played in nearly every African country and represents the diversity of Africa. Individuals of all types enjoy playing this game, royalty and commoners, adults and children, both in cities and villages. Mancala has endured the ages because both past and present culture has been able to enjoy the game in its own special way: as an important family game, a ceremonial right of passage, or a form of recreation among friends.¹ The style of playing pieces and board varies; the wealthy may play on carved ivory boards covered with gold, an egg carton and marbles are an option, and even a few holes in the ground with pebbles as playing pieces will suffice.

As stated above, many different types of Mancala exist throughout the globe; therefore, it is important to establish a set of rules of how to play the game. The following is a list of rules used to play Mancala in this thesis.

1. The board (Figure 1) is placed between the two opponents.
2. The board consists of fourteen pits – six playing pits plus one score pit, the kalaha/cache, per player.
3. Each person has an equal number of playing pieces/pebbles in his or her playing pits (pits #1-#6) to begin the game.
4. To begin play, Player 1 scoops up all the pebbles from any pit on his or her side of the board and moving to the right, drops one pebble into each pit that he or she comes to.
5. If Player 1 comes to his or her cache, Player 1 drops a playing piece in that pit.

6. If Player 1 has more pieces left after dropping one in his or her cache, he or she continues to place the remaining pieces into the playing pits on the opponent’s side of the board.

7. Player 1 never places a playing piece in his or her opponent’s cache. In other words, if Player 1 is depositing pebbles on his or her opponent’s side of the board and reaches the opponent’s cache, Player 1 skips this pit and places the next piece in pit #6 on Player 1’s side of the board.

8. If the last piece in Player 1’s play is placed in Player 1’s own cache, then Player 1 gets another turn.

9. If the last piece is placed in an empty pit on Player 1’s own side of the board, then he or she captures the pieces in the opposite/opponent’s playing pit and transfers all of these pieces to Player 1’s cache along with the piece that landed in the empty pit.

10. Players are not allowed to count the number of pebbles in any pit on the board at any time during the game (players are expected to keep track of the number of pebbles in their heads).

11. The game ends when all of the playing pits on one side of the board are empty. The player with pieces remaining on his or her side of the board gets to put those pebbles into his or her cache.

12. The winner is the player with the most pebbles in his or her cache.
Chapter 2: Determining an Optimal Strategy

My analysis into an optimal playing strategy for the game Mancala began with the most simple scenario, twelve total pebbles, one in each player's playing pit. The 12-piece game proved trivial to analyze (as is evidenced later), so the focus quickly shifted to a twenty-four piece game, two in each player’s playing pit. To begin, a game tree was employed to model a 24-piece game to gain insight regarding potential optimal playing strategies (Figure 2), but the game tree grew fast and became so complex that it was not a reasonable modeling strategy.
Figure 2. This game tree represents possible moves in a 24-piece game of Mancala. The series of branches is similar to moving pebbles from one of the six pits.
A game tree provides a complete enumeration of all possible moves (and thus outcomes) regardless of the number of pebbles used in the game, but as the number of playing pieces increases, so does the complexity of the game tree. Using a game tree for Mancala would provide insight to the status of the board three, four, five moves into the game. As a player, knowing all board possibilities is advantageous because he or she knows the optimal move at any stage in the game.

The game tree for the 24-piece game of Mancala, however, becomes much too complex within the first three moves of the game. As seen in Figure 2, Player 1 has six opening move possibilities. Player 1 can move any of the pebbles in any of his or her six playing pits. On Player 2’s first turn, he or she also has six possible opening moves. After Player 2 moves, Player 1 now has five pits with pebbles, each being a potential move.

In constructing the game tree, it was assumed that no player took his or her opponent’s playing pieces (which limits possible moves) and that the final pebble was not placed in the player’s cache (thus each “branch” of the tree signals one move in a single turn). These assumptions do not accurately represent the game, providing another motive to abandon this method of modeling.

Rather than relying on a game tree, an optimal strategy was formulated by playing the game. Through trial and error many aspects of the game became obvious and these factors were exploited or curtailed when formulating an optimal strategy:

- In a 12-piece game, the player to open should capture an opponent’s pebble during his or her first turn.
• For a twenty-four-pebble game, more than four pebbles should not be accumulated in a single playing pit.

• For a 36-piece or 48-piece games, only one playing pit should house five or more pebbles.

• It is advantageous to earn as many extra moves as possible in a single turn to which maximizes the number of pebbles a player places in his or her cache. This is done by keeping pit #1 open or “pebble-free.”

• It is often better to take the opponent’s pebbles instead of dropping one or two of your pebbles in your cache each turn.

The rationale and motivation behind these statements becomes clear later in the paper during the analysis of each game. Further trends are recognized and developed and consequently included in this list as the complexity of the game increases with the addition of pebbles.

In an attempt to create a reader friendly-paper, Player 1 and Player 2 have been given names. Player 1 is named Kelly and is the focus of the paper. I will investigate the best playing strategy for Kelly. Throughout the analysis, explanations and the reasoning behind Kelly’s choices at each stage in the game are provided. Also, Kelly always moves first in every game. Player 2 is named Jason and unfortunately will be defeated several times in the paper.
Playing Strategies for a 12 Piece Game

A 12-piece game is the most simple scenario possible in Mancala. The analysis of the 12-piece game is simple because the number of total pebbles is small, limiting the number of available moves each player can make when it is his or her turn. If both players make the best move available to him or her when it is their turn, the player that moves first will always win by a final score of seven to five. The essential aspect of this game is that Kelly must capture one of Jason’s pebbles on her first turn. By doing this, Kelly ensures that she will win because Jason is unable to capture the necessary number of Kelly’s pebbles during the game for him to win.

To achieve the capture Kelly opens the game by moving the pebble from pit #1, therefore landing in her cache gaining another move. She should utilize this extra move by capturing Jason’s pebble in his pit #6 by moving pit #2, bringing the number of pebbles in her cache to three. Below a game situation is played out demonstrating the importance of this opening series of moves and Jason’s inability to regain the pebble he lost at the beginning, preventing him from winning.

The board setup to begin a 12-piece Mancala game is shown in Figure 3.

![Board Setup](image)

Figure 3: Starting board for a 12-piece Mancala game.
Kelly moves first. She should move pit #1 (Figure 4), which allows her to gain an extra move in which she can capture Jason’s pebble in his pit #6, by moving her pit #2 (Figure 5).

![Diagram of pits and pebbles]

**Figure 4:** Kelly moves pebbles from pit #1 as her opening move.

\[ \downarrow \]

![Diagram of pits and pebbles]

**Figure 5:** Kelly moves pebbles from pit #2 for her extra move.

There are a few move options that Jason has during his turn, but the end result does not change. Here Jason moved his pebbles in the following sequence, 1→2 (Figure 6).
Kelly is now faces a decision, should she move pit #5 to protect herself from capture, or should she move pit #3 and capture one of Jason’s pebbles? If she decides to protect herself from capture, the game will end in a tie, and is therefore not the best possible move that she can make at this point in the game. Therefore, Kelly should move pit #3 and capture Jason’s pebble (Figure 7). Even though one of her pebbles will be taken by Jason on his next turn (Figure 8), Jason has to take two of Kelly’s pebbles to tie the game and this is not possible.

Jason moves pit #3, capturing one of Kelly’s pebbles (Figure 8).
There is only one possible move that Kelly can make, pit #4, and this conveniently ends the game and Kelly wins seven to five (Figure 9).

While this is only one possible series of moves for a 12-piece game, the outcome of the game will never change; the player to move first will win. The final score will always be seven to five if both players make optimal moves at each turn. If Jason would have moved pit #5 during his second turn (Figure 8) instead of pit #2, the outcome would have been the same; it is impossible for the player moving second to win if the player that moves first captures one of his or her opponent's pebbles on their opening move. So, if playing a 12-piece game of Manacala, one had better hope that they get to move first so...
that he or she can win. A 24-piece game of Mancala is definitely more interesting and this game is examined next.

**Playing Strategies for a 24 Piece Game**

The following is what I believe to be an optimal playing strategy for a 24-piece game of Mancala. A quick overview of the strategy is given, but a deeper analysis and explanation is provided during the play of the games that follow.

In a 24-piece game, the board begins with two playing pieces in each playing pit as seen in Figure 10. As in the 12-piece game, the first move is important. Kelly should move pit #1 as her opening move (Figure 11). On her second turn, she should move as follows, $2 \rightarrow 1 \rightarrow (3 \text{ or } 4)$ (Figures 13-15). These series of moves are standard and should always be followed in the 24-piece game. Moving in this manner sets up the board so that Kelly has the advantage and maintains control of the game. The reasoning behind these moves and the corresponding playing strategy is explained below as the games progress.

Below are two simulations of actual games. In the first game, Jason mirrors the moves of Kelly, illustrating that if the opponent chooses this strategy the game will end in a tie. In the second game, Jason plays an alternative strategy that initially appears fruitful while Kelly plays the same strategy as before and wins the game.
Game 1

Kelly opens the game and Jason mirrors Kelly’s moves. The board setup is shown in Figure 10.

To begin the game, Kelly should move the pebbles from pit #1 (Figure 11). This move does a couple of things. First, it moves the pebbles out of her pit #1 which is beneficial for later moves. Emptying pit #1 as an opening move allows her to control the number of consecutive turns she will gain later in the game. Also, this initial move places three pebbles in the Jason’s pit #6 (Figure 12). Now, during Jason’s turn, if he moves the pebbles in his pit #6, Kelly will be able to take three pebbles out of his pit #5 on her next turn or if Jason chooses to leave these pebbles in pit #6, Kelly can then take three pebbles out of his pit #6. Either move on Jason’s part will allow Kelly to take three of his pebbles on her next turn.
Jason is mirroring Kelly during this game, so he moves pit #1 as well (Figure 12).

On Kelly’s second turn, she should move the pebbles from her pit #2 because she will land in her cache and earn another move (Figure 13). Next, she should move pit #1 because it only has one pebble in it, gaining yet another move (Figure 14). Now, Kelly moves pebbles from pit #3 or pit #4 based on which pit of Jason’s has three pebbles in it (in the game below Kelly moves pit #3). (Jason’s pit #5 and pit #6 cannot have three pebbles in each; it will be one or the other. Jason might have chosen to move the pebbles from pit #5 or pit #6 as his opening move, thus dictating Kelly’s next move. If this situation occurs, there is only one pit available for Kelly to capture pebbles from Jason.)
Another possible scenario is that Jason will choose not to move any pebbles from pit #5 or pit #6 and therefore Kelly can choose to take either of the pebbles in those pits.)

![Diagram of game state]

**Figure 13:** Kelly moves pit #2 and gains an extra move.

\[\downarrow\]

**Figure 14:** Kelly moves pit #1 gaining another move.

\[\downarrow\]

**Figure 15:** Kelly moves pit #3, capturing three of Jason's pebbles from his pit #6.
Jason moved in the following manner during his turn, 2→1→3 (Figure 16 shows the end result).

![Diagram of a board game setup]

By this time, Jason most likely will have taken some of Kelly’s pebbles from her pit #5 or pit #6. So, once Jason has taken his second turn, Kelly’s third turn is not cut and dry. Kelly must move pebbles so that Jason cannot capture them in his future turns. Kelly will only have two or three move choices, so this step is relatively easy. AT THIS POINT, IT IS IMPORTANT FOR KELLY NOT TO MOVE INTO A TRAP SO THAT JASON CAN TAKE ANY OF KELLY’S PEBBLES. Kelly must move defensively until the game is over.

In the current board situation, Kelly must move pebbles from pit #2 (Figure 17). Any other move will allow Jason to gain more pebbles than Kelly. If Jason decides to move and take some of Kelly’s pebbles, Jason does this at a cost because Kelly will be able to capture more than those pebbles back, winning the game. For example, when looking at Figure 16, it appears as though Kelly’s best move would be to move pit #5 and take two of Jason’s pebbles, but if she does this, she sets herself up to lose three pebbles in pit #4.
because Jason would be able to capture these on his next turn. If Jason captures three of Kelly’s pebbles, she will not be able to win because Jason will have captured one more pebble than her.

![Figure 17: Kelly moves the pebbles form pit #2.](image)

Jason moves pit #2 (Figure 18) for the same reasons that Kelly moved her pit #2 in the previous turn (Figure 17).

![Figure 18: Possible board situation for Kelly’s fourth turn.](image)

The first move that Kelly should make on her fourth turn is to move the pebble from pit #1, earning another move (Figure 19). The second move Kelly should make during this turn is to move the pebbles in pit #4 (Figure 20). If Kelly were to move the pebbles from pit #5 instead, then three pebbles would be stacked up in pit #4, thus allowing Jason the opportunity to capture more pebbles than Kelly and he would consequently win the game.
Figure 19: Kelly moves the pebbles from pit #1 and earns another move.

\[
\begin{array}{c|cccccc}
& 1 & 0 & 0 & 2 & 2 & 0 \\
\hline
7 & 0 & 2 & 2 & 0 & 0 & 0 \\
\hline
8 & & & & & & 11
\end{array}
\]

Kelly

Figure 20: Kelly moves pit #4, capturing two of Jason's pebbles in his pit #5.

\[
\begin{array}{c|cccccc}
& 1 & 0 & 0 & 2 & 0 & 0 \\
\hline
7 & 0 & 2 & 0 & 1 & 0 & 0 \\
\hline
8 & & & & & & 11
\end{array}
\]

Jason

Jason leaves the board as seen in Figure 21 by the following series of moves, 1→4.

\[
\begin{array}{c|cccccc}
& 0 & 1 & 0 & 0 & 0 & 0 \\
\hline
11 & 0 & 0 & 1 & 0 & 0 & 0 \\
\hline
11 & & & & & & 11
\end{array}
\]

Kelly

Figure 21: Possible board situation for Kelly's fifth turn.

Now, neither Kelly nor Jason can take each other's pebbles, so the game is over. Both

Kelly and Jason have twelve pebbles in their caches, ending the game in a tie.
If Jason mirrors Kelly’s moves, the worst Kelly can do is tie. Moving first and playing this strategy Kelly will always tie or win.

**Game 2**

Another strategy for Jason was investigated that proved interesting, but was ultimately beaten by the strategy played by above. In this alternative strategy, Jason moves 2→4 as his opening move (Figure 24). On Jason’s second turn, he moves 3→6. All successive moves were dependent on the board. This situation is played out below with Kelly moving pit #1 on her first turn and moving 2→1→(3 or 4) on her second turn.

Kelly opens the game and Jason does not mirror Kelly. The initial board setup is shown in Figure 22.

![Figure 22: Starting board for a 24-piece Mancala game.](image)

Kelly moves pit #1, which will allow her to control the board and the number of pebbles entering her cache later in the game (Figure 23).
Jason moves as mentioned above, 2→4 (Figure 24).

Kelly now moves in the following manner: 2→1→3, following the strategy of depositing as many pebbles as possible into her cache and then capturing as many pebbles as possible from Jason without jeopardizing future board position (Figures 25-27).
Figure 26: Kelly moves pit #1, earning another move.

Figure 27: Kelly moves pit #3 and captures three of Jason’s pebbles from his pit #6.

Now, Jason is stuck because he cannot capture any of Kelly’s pebbles. Jason does not want to move the pebbles from pit #1 because he would be giving pebbles to Kelly, nor does he want to move pit #3 and increase the number of pebbles in pit #1. Jason has already lost the game, but one possible scenario is played out below to demonstrate the result.

As mentioned above, Jason is stuck and moves pit #5 as a last resort (Figure 28).
At this point, Kelly should move the pebbles from pit #6 (Figure 29) to prevent a future capture (It is possible that Jason could empty his pit #1 and somehow take these pebbles).

It is always wise to move so that your pebbles are “ahead” of your opponent’s pebbles, preventing any possibility of capture by the opponent.

Jason now has a variety of options. He can move pit #4 to stall the game and avoid depositing pebbles on Kelly’s side of the board. Alternatively, Jason could move pit #3, giving Kelly a pebble and loading pit #1 with four pebbles. Either way, Jason is in a world of hurt and is going to lose to Kelly.
Jason moves pit #4 because it is the best move that he can make at this point in the game (Figure 30).

![Diagram of board with pits labeled Jason and Kelly with numbers 0 to 3 and 5 to 7, and pebbles in pits 0 to 1 and 3 to 1.](image)

Figure 30: Possible board situation for Kelly’s fourth turn.

Kelly now moves pit #3 (Figure 31). This stalls the game, forcing Jason to deposit pebbles on Kelly’s side of the board during his next turn.

![Diagram of board with pits labeled Jason and Kelly with numbers 0 to 3 and 5 to 7, and pebbles in pits 0 to 1 and 3 to 1.](image)

Figure 31: Kelly moves the pebbles from pit #3 stalling the game.

Jason moves pit #1 (Figure 32).
Figure 32: Possible board situation for Kelly’s fifth turn.

Kelly moves in the following fashion, 1→2→1→6 on her fifth turn (Figure 33-36). By moving in this fashion, she is depositing several pebbles into her cache.

Figure 33: Kelly moves the pebbles from pit #1 gaining an extra move.

Figure 34: Kelly moves pit #2, gaining an extra move.
Figure 35: Kelly moves pit #1, earning another move.

Figure 36: Kelly moves the pebble from pit #6.

Jason is forced to move pit #3 (Figure 37).

Figure 37: Possible board situation for Kelly’s sixth turn.

At this point, Kelly should move pit #6, then neither Kelly nor Jason can capture each other’s pebbles, and therefore the game is over. Kelly wins sixteen to eight.
For a 24-piece game of Mancala, it is effective to move the pebbles from pit #1 as the player’s opening move and then follow with the 2→1→(3 or 4) combination on the player’s second turn. This guarantees that the worst possible scenario, with both players moving optimally, is the game will result in a tie.

**Playing Strategies for a 36 Piece Game**

In a 36-piece game, the board begins with three playing pieces in each playing pit as seen in Figure 38. A quick comparison between the 24-piece and 36-piece games is given, followed by an overview of strategic moves, with an analysis and explanation provided in the context of the games below.

When comparing the 36-piece game to the 24-piece game, there are obvious differences. First, it is much more difficult to set up your side of the board so as to capture your opponent’s pieces. In other words, in the 24-piece game Kelly could easily make moves to setup her side of the board that was conducive to depositing pebbles in her cache as well as capturing some of Jason’s pebbles. In the 36-piece game, this is much more difficult to do, especially after the first two turns, namely because the number of pebbles in each pit is large (five or six). The reason this increase in pebbles poses a problem is because it becomes difficult to keep track of the number of pebbles Kelly has in her playing pits and plan setup moves that require her to move three and four pits ahead. Another difficulty is that Kelly often will have too many pebbles in her playing pits to land exactly in her cache and earn another move during one turn, instead she will often have to deposit multiple pebbles on Jason’s side of the board.
When playing this game, Kelly must move pebbles so that Jason cannot capture her pieces on his first turn. Therefore, Kelly must move the pebbles in pit #1 as her opening move (Figure 39). This deposit results in a total of three pebbles in Jason’s pits #5 and #6. Now, Jason cannot capture any of Kelly’s pebbles on his first turn. This opening move is essential and sets up the rest of the game. Once Jason has moved, Kelly has some options. After exploring each of these possible moves (the possibilities are moving pit #3, 4, 5, or 6), Kelly should move the pebbles in pit #3. By moving the pebbles in pit #3, Kelly gains an extra move during her second turn. She then moves pit #1, making another deposit in her cache and finally moves pit #4 and the rationale becomes clear as to why later in the analysis. In playing by this strategy, Kelly will have a pileup of pebbles in pit #2, but this is fine because it will never be broken (if the pileup is broken, Kelly would be depositing a large number of pebbles into Jason’s playing pits). The key in this strategy is that you can build up the number of pebbles in one pit, but you cannot have more than one pit with a stack of five or more.

Kelly’s opening moves are as follows:

Turn #1: 1

Turn #2: 3→1→4

The moves that Kelly makes after her second turn are dependent upon the board situation. Below, two games are played out demonstrating different board setups and possibilities and the best move that Kelly can make in these situations. The two different games illustrate how one move can dictate whether Kelly wins or loses. The two playing options explored occur on Kelly’s second turn. If Kelly moves 3→1→6 on her second turn, she loses (Figures 41-43). If Kelly moves 3→1→4 on her second turn, she wins

29
(Figures 59-61). It was shown in the 24-piece game that when Jason mirrors Kelly the game ends in a tie. The result is identical in the 36-piece game and therefore will not be demonstrated again.

**Game 1**

The board setup to begin a 36-piece Mancala game is shown in Figure 38.

![Starting board for a 36-piece Mancala game](image)

**Kelly**

Figure 38: Starting board for a 36-piece Mancala game.

Kelly opens the game by moving pit #1 (Figure 39).

![Kelly moves the pebbles from pit #1](image)

Figures 39: Kelly moves the pebbles from pit #1.

For the opponent, there is not an obvious opening move. Jason will never set himself up to be captured unless that is the only option. Here Jason moves pit #2 (Figure 40). Any
of the six playing pits could have been moved, but pit #2 was chosen to illustrate how one move can dictate the outcome of the game.

![Diagram of playing pits]

Kelly now moves in the following manner: 3→1→6 (Figure 41-43). Following this series of moves, Kelly will only accumulate pebbles in pit #2.

![Diagram of playing pits after Kelly's move]

Kelly moves the pebbles from pit #3 gaining an extra move.
<table>
<thead>
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<th>Kelly</th>
<th>Jason</th>
</tr>
</thead>
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<td>3</td>
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<td></td>
</tr>
</tbody>
</table>

**Figure 42:** Kelly moves the pebbles from pit #1 earning another move.

<table>
<thead>
<tr>
<th>Jason</th>
<th>Kelly</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0</td>
</tr>
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**Figure 43:** Kelly moves the pebbles from pit #6.

Jason moves pit #4 as his next move, creating the board setup seen in Figure 44.

<table>
<thead>
<tr>
<th>Jason</th>
<th>Kelly</th>
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**Figure 44:** Possible board situation for Kelly’s third turn.

Kelly must now move pit #3 because she needs to stall so that Jason is forced to place pebbles in Kelly’s playing pits (Figure 45). Note that Kelly has a build up of pebbles in
only one playing pit, pit #2, while Jason has a build up of pebbles in four playing pits.

Kelly should take advantage of Jason’s growing pits by moving pit #3 to stall the game (Figure 45). This will eventually force Jason to place his pebbles in Kelly’s playing pits.

![Figure 45: Kelly moves the pebbles from pit #3.]

Here Jason has chosen to deposit pebbles in his cache, while building up the number of pebbles in his pit #1. This move will allow Jason to completely wrap around the board on his next turn and take the pebbles that Kelly is accumulating in her pit #2. Jason moved 4→5→2→3 leaving the board as seen in Figure 46.

![Figure 46: Possible board situation for Kelly’s fourth turn.]

At this point, there is no move Kelly can make that will allow her to win the game.

Regardless of which pit Kelly chooses to move, she will lose. If Kelly chooses to move
pit #2 (Figure 47), Jason will have enough pebbles on his side of the board to stall, forcing Kelly to put more of her pebbles on Jason’s side of the board. If Kelly attempts to stall and move either pits #5 or #6, Jason could take Kelly’s pebbles in pit #4. Moving pit #4 allows Jason to collect pebbles in his pit #1, eventually gaining enough pebbles in this pit to capture Kelly’s pit #2 by wrapping around the board. One possible solution is played out below.

![Diagram of board setup with pebbles]

**Figure 47:** Kelly moves the pebbles from pit #2.

Jason moves pit #2 to stall (Figure 48).

![Diagram of board setup with pebbles]

**Figure 48:** Possible board situation for Kelly’s fifth turn.

Kelly should now try to stall by moving pits #5 and #6. Moving pit #5 will prevent immediate capture (Figure 49).
Figure 49: Kelly moves the pebbles from pit #5.

Jason moves pit #6 to stall (Figure 50).

Figure 50: Possible board situation for Kelly’s sixth turn.

Kelly moves pit #6 as a last resort in an attempt to stall (Figure 51).

Figure 51: Kelly moves the pebbles from pit #6.

Jason moves pit #3 and captures the pebbles in Kelly’s pit #5 (Figure 52).
Now, Kelly must move pit #1 (Figure 53) which results in an extra move forcing her to move pit #4 (Figure 54) on this extra move.

Jason

Kelly

Figure 54: Kelly moves the pebbles from pit #4.

Jason continues to stall by moving pit #4 (Figure 55).
At this point, the game is over because Kelly is unable to capture the necessary number of Jason’s pebbles to win or tie. Kelly loses 10-26. Notice the difference in the outcome of the game when Kelly moves pit #4 on her second turn versus pit #6.

**Game 2**

In this game, Kelly wins by altering the sequence of moves she chooses to make on her second turn. Here, Kelly chooses to move 3→1→4 (Figures 59-61) on her second turn. The initial board setup to begin the game is shown in Figure 56.

Kelly opens game 2 as she did in game 1, by moving pit #1 (Figure 57).
Figure 57: Kelly moves the pebbles from pit #1.

For this game, Jason opens as he did before and moves pit #2 (Figure 58).

Figure 58: Possible board situation for Kelly's second turn.

Kelly now moves in the following manner: 3→1→4 (Figures 59-61) verses 3→1→6 (Figures 41-43) as she did above and the outcome is drastically different. This is because Jason is now unable to capture any of Kelly's pebbles on his next turn.

Figure 59: Kelly moves the pebbles from pit #3 earning another move.
Figure 60: Kelly moves the pebbles from pit #1 gaining an extra move.

Figure 61: Kelly moves the pebbles from pit #4 capturing four of Jason’s pebbles.

Here Jason moves 3→5 and leaves the board as seen in Figure 62.

Figure 62: Possible board situation for Kelly’s third turn.
At this point, Kelly moves pit #3 to stall the game (Figure 63). She is hoping that Jason will eventually have to break up his playing pits housing several pebbles and deposit these pebbles on her side of the board.

![Figure 63: Kelly moves the pebbles from pit #3.](image)

Here Jason has decided to move 2→4→3 because he is able to gain two extra moves and deposit two pebbles into his cache while keeping his pebbles on his side of the board and preventing capture by Kelly on her next turn. The resulting board setup for Kelly’s fourth turn is shown in Figure 64.

![Figure 64: Possible board situation for Kelly’s fourth turn.](image)

Kelly moves pit #5 to stall the game which will force Jason to deposit pebbles on her side of the board on his next turn (Figure 65).
Figure 65: Kelly moves the pebbles from pit #5.

Jason moves pit #2 (Figure 66).

Figure 66: Possible board situation for Kelly’s fifth turn.

Kelly now moves pit #4 again to stall (Figure 67).

Figure 67: Kelly moves the pebbles from pit #4.
Jason moves pit #1, it is his only option. During this turn, he captures seven of Kelly’s pebbles (nine total, two originated with him) that are housed in her pit #2 by wrapping around the board (Figure 68).

![Figure 68: Possible board situation for Kelly’s sixth turn.]

Yes, Jason was able to capture seven of Kelly’s pebbles, but Kelly still wins. The sequence of moves for Kelly’s sixth turn is as follows: 1→3→1→2. The last move captures Jason’s last pebble and the game is over. Kelly wins 22-14.

These two different games demonstrate how vital a single move can be in the final outcome of the game. Moving pit #4 (Figure 61) verses pit #6 (Figure 43) on Kelly’s second move was the difference between winning and losing. Also, this one move was not obvious during Kelly’s second turn, illustrating how difficult it can be to determine whether or not a move is optimal at this level.

**Playing Strategies for a 48 Piece Game**

It is difficult to derive a series of playing moves in a 48-pebble game that would always lead to a victory by Kelly. Once the game progresses past the 24-pebble level, the
moves that Kelly chooses become more dependent upon what choices Jason is making as was witnessed in the 36-piece games above. In a 48-piece game, the board begins with four playing pieces in each playing pit as seen in Figure 69.

At the 48-pebble level, the game is quite complex for a variety of reasons. First, it becomes difficult to plan future moves. This is because there are too many pebbles in each playing pit to keep track of in Kelly’s head (because counting the number of pebbles in a pit is not allowed) and quite often Jason’s move will add pebbles on Kelly’s side of the board, consequently spoiling the move Kelly was intending to do on her next turn. When Jason adds pebbles to Kelly’s pits, it is highly unlikely that Kelly will be able to either take pebbles from Jason by landing in an open pit or earning another turn by placing the last pebble in her cache.

A second element of the game that proves to be an influential factor is playing pits become loaded with pebbles. During the middle of a game, it is not uncommon for both players to have at least two playing pits containing more than eight pebbles. A player does not want multiple pits containing a large number of pebbles because eventually he or she will have to move one of these pits. When one these pits is moved, the player will then end up giving his or her opponent the pebbles that are in this pit. Thus, it is imperative that players make a conscious effort to limit the number of pebbles contained in each pit. The exception to this rule is that a player can have one pit with many pebbles. I have found that the most effective pit to house several pebbles is pit #2. Breaking up pits with large numbers of pebbles early in the game, except for pit #2, is essential to winning the game.
If Jason is caught with multiple pits housing a large number of pebbles, Kelly should attempt to force Jason to “break-up” these pits, causing him to deposit these pebbles into her playing pits. To do this, Kelly should move pebbles from pits that have the least number of pebbles in them. In other words, Kelly should make a series of moves where she only moves one or two pebbles at a time for two, three, or four turns in a row so that eventually Jason will have to move the pebbles from a pit with many playing pieces. This is a “stall” tactic. Moving pebbles one or two at a time allows Kelly to have several turns. The hope is that Jason will “run-out” of these types of moves before Kelly and will thus have to opt for moving a pit containing a large number of pebbles, and conveniently depositing these pebbles on Kelly’s side of the board.

Below two games are started demonstrating the complexity of the 48-piece game and how quickly the number of pebbles in the playing pits grow. As always, Kelly opens the game. Also, emptying pit #1 as an opening move has proved effective in both the 24-piece and 36-piece games and is in turn recommended for the 48-piece game.

**Game 1**

The board setup to begin a 48-piece Mancala game is shown in Figure 69.

![Figure 69: Starting board for a 48-piece Mancala game.](image-url)
Kelly moves pit #1 to begin the 48-piece game because it has proved effective in the 24 and 36-piece games (Figure 70).

![Diagram of a game board]

Figure 70: Kelly moves the pebbles from pit #1.

Jason moves 5→6 as his series of opening moves (Figure 71).

![Diagram of a game board]

Figure 71: Possible board situation for Kelly’s second turn.

For her second turn, Kelly moves 4→1→4 (Figures 72-74). By moving in this fashion, she is able to gain extra moves during this turn while depositing two pebbles in her cache.
Figure 72: Kelly moves the pebbles from pit #4 gaining an extra move.

\[
\begin{array}{ccccccc}
1 & 6 & 6 & 6 & 7 & 1 & 0 \\
4 & 4 & 0 & 5 & 5 & 1 & \ \\
\end{array}
\]

Kelly

Figure 73: Kelly moves the pebbles from pit #1 earning another turn.

\[
\begin{array}{ccccccc}
1 & 6 & 6 & 6 & 7 & 1 & 0 \\
4 & 4 & 0 & 5 & 5 & 0 & \ \\
3 & \ \\
\end{array}
\]

Jason

Figure 74: Kelly moves the pebbles from pit #4.

\[
\begin{array}{ccccccc}
1 & 6 & 6 & 6 & 7 & 1 & 0 \\
4 & 0 & 1 & 6 & 6 & 1 & \ \\
3 & \ \\
\end{array}
\]

Jason

At this point, Jason will most likely move pit #5 which Kelly can counter by moving pit #4. After this series, Jason is then forced to move a pit housing several pebbles and deposit them on Kelly’s side of the board. Now, Kelly should move pit #3 to
prevent a build up of pebbles in this pit, which allows for more capture opportunities and omits the potential of having to move a pit with many pebbles, giving unnecessary stones to Jason.

**Game 2**

The board setup to begin a 48-piece Mancala game is shown in Figure 75.

![Figure 75: Starting board for a 48-piece Mancala game.](image)

Kelly moves pit #1 to begin the 48-piece game because it has proved effective in the 24 and 36-piece games (Figure 76).

![Figure 76: Kelly moves the pebbles from pit #1.](image)

Jason has decided to move 5→4, leaving the board as seen in Figure 77.
Kelly should now move $5 \rightarrow 1 \rightarrow 6$ (Figures 78-80). This series of moves allows Kelly to deposit pebbles in her cache and capture some of Jason’s pebbles.
Figure 80: Kelly moves the pebbles from pit #6 capturing five of Jason’s pebbles.

Kelly has now forced Jason to deposit pebbles on her side of the board. Kelly is able to move so that the game will be stalled further forcing Jason to make unwanted deposits in Kelly’s playing pits, allowing Kelly to win the game.

Chapter 3: Conclusion

Mancala is a great game due to its varying levels of complexity. Investigating the 12-piece game was trivial; the player to move first wins. Analyzing a 24-piece game is quite tractable; the number of pebbles in each pit is small and easy to track in one’s head thus allowing the player to plan future moves. A 36-piece game becomes more difficult, but trends are recognizable and certain strategies can be employed easily by players. A 48-piece game is heinous to examine; determining possible playing strategies is tough due to the advanced level of difficulty that arises with the addition of playing pieces.

The 48-piece game is standard in America for several plausible reasons. First, it is customary that players are not allowed to count their pebbles prior to moving, rather players must mentally tabulate the number of pebbles he or she has as well as how many
the opponent has in each playing pit. This becomes extremely difficult as more pebbles are added to the game, as witnessed in the 48-piece games above. Second, it is extremely difficult to play a game with 48-pieces and have a single winning strategy. Almost all of the moves a player makes in a game are dependent upon what a player’s opponent is doing on his or her side of the board. Often times, a player can plan out a series of two moves, but the plays of his or her opponent ruin that planned strategy by depositing pebbles on the opposite side of the board.

Playing 48-piece Mancala is both fascinating and challenging. Determining an optimal strategy for games that reach this level of complexity is extremely difficult (and quite time consuming) because the move that each player makes is dependant upon the opponent’s play. There are, however, definite tendencies in the game that allow players to make moves exploiting these trends. For example, allowing only one pit to become built up with many pebbles and always keeping pit #1 empty are elements to the game that should be exploited. What becomes obvious in the 48-piece game is that players are unable to optimize moves globally. In other words, players have to focus on what is the best move that I can make now that will help me win verses what is the best move that I can make now that will help me four or five turns later to ultimately win the game.

The analysis has assisted in determining that a rational move in Mancala is one that

a) deposits stones in one’s cache
b) captures opponent’s stones
c) stalls the game,
d) forces the opponent to deposit stones on one’s side of the board.
If players attempt to follow these guidelines defining a rationale and best move, they have a good chance of winning the game.

As illustrated in the 36-piece games, a single move can determine the eventual outcome. As a player, the game depends upon what moves your opponent has made and then countering with the best move or series of moves possible. There is not a single optimal strategy beyond the 24-piece game. To win at higher levels of the game requires thought, recognizing and following the mentioned trends, and possibly some errors on your opponent’s part.
References

1. 12 Feb 2003 <www.cmi.k12.il.us/Urbana/projects/AncientCiv/africa/Mancala.html>
