Analysis of Magnetohydrodynamic Simulations of Convective Forces on Buoyant Flux Tubes in the Sun

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Analysis of Magnetohydrodynamic Simulations of Convective Forces on Buoyant Flux Tubes in the Sun

By

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Submitted in partial fulfillment of the requirements for graduation with honors to the Department of Mathematics, Engineering, and Computer Science

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This thesis for honors recognition has been approved for the Department of Mathematics, Engineering, and Computer Science.

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Analysis of Magnetohydrodynamic Simulations of Convective Forces on Buoyant Flux Tubes in the Sun

Abstract

Buoyant flux tubes of magnetic energy in the sun rise through powerful and turbulent convection to penetrate the sun’s photosphere and produce sunspots. The question studied in this research paper is: How do magnetic flux tubes rise through this violent convection to the solar surface without being destroyed? We analyze data in 128*128*300 arrays generated from supercomputer simulations of three-dimensional magnetized gas to investigate the sensitivity of flux tube survival time to different parameters: the magnetic field strength of the tubes, the viscosity of the solar fluid, and the magnetic diffusivity. We also calculate the standard deviation of magnetic energy density within the arrays to set a scale that quantitatively describes flux tube coherence. Both quantitative and qualitative measures indicate that increased magnetic field strength and increased viscosity greatly expand the time of the flux tubes’ survival, whereas increased magnetic diffusivity minutely inhibits flux tube survival time.
Introduction

First Observations of Solar Magnetic Phenomena

Our sun has always been an enigma. Scientists have long sought to understand the puzzles of the sun’s magnetic phenomena that until recently have not been thought to affect human activity. One such observable magnetic structure is the sunspot. In 1610, Thomas Harriott was the first to record observations about sunspots. Several centuries later, inquisitive scientists began to take interest in the sun’s magnetic activity. Schwabe observed in 1843 that sunspots emerged in decreasing frequency over a period of eleven years, after which time their frequency of emergence would increase, and in 1858 Carrington published his findings that sunspots would appear closer to the equator as the eleven-year cycle progressed (Figure 1: see page 24 for a table of figures). This is now known as Spörer’s law of sunspot latitudes.

Hale (1908) was the first to find that sunspots are truly magnetic structures, recording that sunspots had magnetic fields up to 3,000 Gauss, and he noted that sunspots occurred in pairs aligned along slightly tilted east-west latitudes. He also discovered that one sunspot of a pair would have a magnetic field pointing radially outward while its counterpart would have a magnetic field pointing radially inward. Conversely, all the easternmost sunspots of a pair within the northern hemisphere had the same polarity as the westernmost sunspots of a pair within the southern hemisphere, and that the polarities in these regions would reverse after each eleven-year cycle. These patterns are now known collectively as the Hale polarity laws (Figure 1) (Hale et al. 1919).
**Properties of the Solar Interior Contributing to the Solar Dynamo**

The sun is a turbulent mass of hydrogen and helium plasma. Intense heat, generated by the fusion of protons within the core of the sun, must escape to the cooler regions of space. First, energy in the form of photons slowly diffuses outward through the radiative zone that surrounds the sun’s core. However, the outer 30% of the sun is made of opaque gases that do not permit much heat to radiate out into space. Hence the gases must transport heat from the outer 30% of the sun to outer space by means of powerful and turbulent convection with a Reynolds’ Number of about $10^{12}$ (Cline 2003). Additionally, the sun’s intense heat provides electrons with enough energy to dissociate from the hydrogen and helium gases that comprise the sun so that positively charged nuclei and negatively charged electrons float freely from each other. This ionized gas is called plasma. Since the electrons are dissociated from nuclei, they freely move within the plasma, so solar fluid conducts electricity very well. As a result of this attribute, the sun’s magnetic field freezes into ionic plasma, so as the solar plasma moves, the magnetic field held within that plasma moves as well. These properties have led us to conclude that magnetohydrodynamic simulations of solar material may accurately depict what is truly going on within the convective region of the sun.

The core and the radiative zone of the sun exhibit solid-body rotation. They rotate with the same angular velocity. The convective region, however, exhibits differential rotation: Solar material near the equatorial surface of the sun makes a complete revolution around the sun’s axis in about 25 days, whereas solar material near the poles requires about 33 days to revolve around the sun’s axis. This is known as the $\omega$-effect. At the interface of the convective zone and the radiative zone is an intense
shear region, called the solar tachocline, a very thin layer where the velocity of the solar fluid changes drastically. The sun begins its magnetic cycle with a poloidal magnetic field. Cowling (1953) suggested that this shear causes the dipolar field to be drawn out, wrapping the magnetic field around the sun to form toroidal (latitudinal) magnetic fields (Figure 2). It takes approximately three years for a poloidal magnetic field line to be wrapped five times around the northern and the southern hemispheres. As differential rotation stretches the magnetic field, the field’s magnitude increases, just as stretching a spring increases the energy of the spring system.

Babcock, another contributor to our understanding of the sun’s inner workings, discovered that as the eleven-year cycle of the sun progresses, poloidal magnetic fields formed by coalescence of toroidal fields, which he calls bipolar magnetic regions (BMRs), can form at lower and lower latitudes, accounting for Spörer’s law (Babcock 1961).

**Solar Magnetic Flux Tubes**

Parker (1954) found that in an electrically conducting atmosphere, such as plasma, a horizontal magnetic flux tube will tend to be less dense than surrounding unmagnetized material, and so it becomes buoyant. And cyclonic fluid rotations, such as what those that are caused by the Coriolis effect, would twist toroidal flux tubes. Such occurrences would account for much of the solar activity that emerges through the photosphere. Additionally, Parker (1955) found that these tubes would coalesce, regenerating the poloidal magnetic field. Thus, we have a mechanism for a cyclic generation of poloidal and toroidal magnetic fields, and we have a means by which
magnetized plasma could rise and emerge from the surface of the photosphere, producing sunspots.

In the twentieth century, mathematicians and astrophysicists have proposed feasible mechanisms that adequately predict and describe many of the sun's magnetic phenomena. One of the most recent problems for astrophysicists has been to determine how a solar flux tube can rise through the strong turbulence in the sun's convective zone and emerge to produce sunspots without being shredded to pieces. The convective forces are so strong that only the strongest buoyant magnetized material can emerge through the photosphere. One approach to the problem of visualizing just how these flux tubes rise through the convective zone and how strong they must be to do so is to analyze computer simulations of magnetic flux tubes in a convective region. Once computers generate data sets, the sets can be analyzed to see what parameters must exist for a flux tube to be able to survive. Many factors determine whether a flux tube can survive in the convective zone. For example, if the viscosity of the solar fluid is decreased, the violence of convection increases, tearing apart the magnetic flux tube within a shorter time. If the magnetic energy density of the flux tube is increased, the tube becomes more rigid so may hold up under stronger convective or buoyant forces.

*Clarification of the Problem*

The question to be studied in this research is: How do magnetic flux tubes successfully rise to the solar surface through this convection without being destroyed? This problem is currently being researched by several different groups of astrophysicists (Abbett, Fan, & Bercik 2004; Fan, Abbett, Fisher 2003; Cline, Brummell, Cattaneo 2004).
Description of the Data Sets

In this research project, we analyzed data from supercomputer simulations of three-dimensional magnetized gas. These data sets are magnetohydrodynamic simulations of magnetic flux tubes from a model designed by Cline (2003). Each set contains $128^2 \times 300$ arrays that describe the velocity, magnetic field, temperature, and density of the magnetized gas at each point at different steps in time. One time step is representative of $1/10,000$ of a sound-crossing time, and we have data for every 5,000 time steps, starting at $t = 0$. Time units are irrelevant, since our task is not to determine how long these tubes last relative to some reference frame. We are comparing the robustness of magnetic flux tubes relative to each other, so we need only use time frames for each case that are consistent. For example, we can say that at $t = 20,000$ the flux tube in Case 108 has existed in the solar environment for as long as the flux tube in Case 111, and we can compare how one tube holds up better than the other. With the time-dependent arrays, we can track the changes of the flux tubes as magnetic diffusion, convection, and buoyancy shred the tubes while the magnetic fields exert tension on the tubes to hold them together. We use the Cartesian coordinate system to describe the position of each point in the array. In this case, the $x$-direction is parallel to the sun’s longitudes. The $y$-direction is parallel to the sun’s latitudes, and the $z$-direction is pointing directly to the core of the sun, so modeled flux tubes are parallel to the $y$-axis, since they have been stretched toroidally by differential rotation. And as time passes, buoyant flux tube should rise from $z = 150$ to $z = 0$. See Figure 3 for an example a tube.
Flux Tube Coherence

The first problem we must encounter is to define mathematically what flux tube coherence is. Although we can give a general estimate of the time a magnetic flux tube remains coherent by qualitatively observing the magnetic energy density of a tube, such a procedure would be time-consuming and tedious. We would have to load individual sets, examine three-dimensional schemes of the magnetic energy density at different times, and judge when we think the tube can no longer be called a tube. It would be much simpler to have an equation at hand that receives as input all the parameters that go into computing the data sets, and gives as output how long a flux tube would remain coherent. Besides, qualitative analysis leaves room for judgments based on opinions of what a coherent flux tube is, and although developing a mathematical model of flux tube coherence would require some qualitative observation, it would be best to have a rigorous method for calculating just how coherent a tube is based on the distribution of magnetic energy density.

To create a visual image of what makes bits of magnetic energy density be aligned in such a fashion as to be able to be considered a tube, we must ask what the attributes of a tube are. Possibly the best way to convey what a coherent tube looks like is to show a picture of one from one of the data sets that we are analyzing. Figure 3 is an example of a computer-generated magnetic flux tube before any convective forces have acted on it. This structure is pretty regular: The magnetic energy density is clustered in a cylindrical form and the center of the structure is apparent. Once forces begin to act on the tube, it will twist, contort, and eventually be torn to shreds. How long the tube lasts depends on certain parameters that are programmed into the computer.
**Parameters**

Some parameters describe the robustness of a magnetic flux tube, and others describe the vigor of forces that would act to tear apart the flux tubes. See Cline (2003) for further details on these parameters. One important factor is the strength of the tube's magnetic field $\alpha$. A magnetic flux tube with a large $\alpha$ has a strong magnetic field, and so it would be expected that such a tube would sustain convective and diffusive forces and remain coherent for a longer period of time than a tube in a similar environment with a small $\alpha$. The flux tube in Case 108, for example, has $\alpha = 0.024$, whereas the flux tube in Case 111 has $\alpha = 24$. Additionally, all other parameters in Case 108 are equal to those in Case 111. Figure 4 provides a visual comparison of the flux tube from Case 108 to the flux tube from Case 111 at $t = 25,000$.

Another parameter to consider is the thermal dissipation parameter $C_k$. Although typically used to describe the rate at which heat dissipates, $C_k$ may also be conjoined with other parameters to indicate the magnetic diffusivity and the viscosity of the array. The magnetic diffusivity is described by equation 1:

$$ \zeta \cdot C_k \cdot \nabla^2 \overrightarrow{B} $$

(Eq. 1)

where $\zeta$ is the magnetic Prandtl number, $\nabla^2$ (del-squared) is $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, and $\overrightarrow{B}$ is the vector describing the magnitude and direction of the magnetic field at each point in the array. Since equation 1 describes the magnetic diffusivity, or the rate at which magnetic energy density is spreading out, a tube in an environment with a small $\zeta$ or $C_k$ would tend to remain more magnetically concentrated and would be expected to hold up better than a tube with a large $\zeta$ or $C_k$ whose magnetic energy density would spread out.
more quickly. Case 111 and Case 125 can be compared to see the effects of $\zeta$ on the evolution of the magnetic structures. Figure 5 provides a visual comparison of the flux tube from Case 111 to the flux tube from Case 125 at $t = 25,000$.

$C_k$ is also directly proportional to the viscosity of the solar fluid. The viscous force on the solar fluid is described by equation 2:

$$P_r \cdot C_k \cdot \nabla^2 \mathbf{v}$$  \hspace{1cm} (Eq. 2)

where $P_r$ is the Prandtl number and $\mathbf{v}$ is the vector describing the velocity of the solar fluid at each point in the array. $P_r$ is useful for comparing the impacts of viscosity on flux tube life. Since the velocity describes the motion of the fluid, it is inversely related to the viscosity of the fluid. For example, the viscosity of molasses is high compared to that of water, so we would expect water to move faster than molasses under the same conditions. But the solar fluid is much less viscous than water.

If $C_k$ were increased, we would expect as a consequence that the viscosity would increase, so the solar fluid’s velocity would decrease, and a flux tube would last longer. Here, we have conflicting factors. Since the product of $C_k$ and $\zeta$ describes the magnetic diffusivity, a case with a larger $C_k$ would be expected to have a greater magnetic diffusivity, resulting in a shorter flux tube life. But it would also be expected to have a higher viscosity, resulting in a longer flux tube life.

Several other parameters describe the solar environment which are not analyzed: the polytropic index, the temperature contrast index $\theta$, and the ratio of specific heats $\gamma$. The polytropic index describes the stratification of density in the convective zone. Since gas molecules in an atmosphere support the weight of gas molecules above them (much like the earth’s atmosphere), gas should be denser near the bottom of the convective zone.
and less dense near the top of the convective zone. The polytropic index functions to hold the array in hydrostatic equilibrium. In other words, a polytropic index is selected so that the density of gas at any given altitude is sufficient to support the gas above it. Theta (θ) describes the temperature contrast across the convective layer. Since heat is radiating from the core of the sun outward, the temperature should be greater at the bottom of the convective zone than at the top. In the cases we analyze, the temperature contrast is assumed to be linear. Gamma (γ) is a ratio of specific heats that describes the buoyancy of the flux tube determined by the adiabatic expansion of gas. Although changing these three parameters would affect the evolution of a buoyant flux tube rising to the surface of a convective zone, the values of these parameters are chosen to make the simulation stable (as in the case of the polytropic index or theta) or their value has been empirically determined (as in the case of gamma), so we do not analyze their effects.

Table 1 lists the simulation parameters. Cases 108, 111, 123, and 124 differ only in their α values. Cases 125 and 126 differ only in their values of ζ. Cases 111 and 124 differ in $P_r C_k$ but not significantly in $ζ C_k$, so they can be compared to analyze the effects of viscosity. We can compare the effects of changing $C_k$ with Cases 140 and 142.

<table>
<thead>
<tr>
<th>Case</th>
<th>α</th>
<th>ζ</th>
<th>$C_k$</th>
<th>Prandtl Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>108</td>
<td>0.024</td>
<td>0.001</td>
<td>0.4899</td>
<td>0.1</td>
</tr>
<tr>
<td>111</td>
<td>24</td>
<td>0.001</td>
<td>0.4899</td>
<td>0.1</td>
</tr>
<tr>
<td>123</td>
<td>48</td>
<td>0.001</td>
<td>0.4899</td>
<td>0.1</td>
</tr>
<tr>
<td>124</td>
<td>96</td>
<td>0.001</td>
<td>0.4899</td>
<td>0.1</td>
</tr>
<tr>
<td>125</td>
<td>24</td>
<td>0.0001</td>
<td>0.4899</td>
<td>0.1</td>
</tr>
<tr>
<td>136</td>
<td>96</td>
<td>0.002</td>
<td>0.2499</td>
<td>0.1</td>
</tr>
<tr>
<td>140</td>
<td>120</td>
<td>0.002</td>
<td>0.2499</td>
<td>0.1</td>
</tr>
<tr>
<td>142</td>
<td>120</td>
<td>0.002</td>
<td>0.06928</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 1: A table of parameters. Cases 108, 111, 123, and 124 differ only in their α values, as do cases 136 and 140. Cases 111 and 125 differ only in their values of ζ. Because all of the cases have the same Prandtl number, we compare the effects of changing viscosity with Cases 125 and 136, since the product of ζ and $C_k$ are nearly equal. We compare the effects of changing both viscosity and magnetic diffusivity, since cases 140 and 142 differ only in their $C_k$ values.
Methods of Analysis

First Model: Numerical Differentiation

The first goal is to find the center of the flux tube. We start by evaluating the simplest flux tube in the data set—Figure 3. Because we are working with magnetic energy density values at discrete intervals, we apply a centered finite-divided method of numerical differentiation to the data array,

\[ f'(x_i) = \frac{-f(x_{i+2}) + 8f(x_{i+1}) - 8f(x_{i-1}) + f(x_{i-2})}{12h} \]  

(Eq. 3)

calculating the change in magnetic energy density with changes in the x- and z-directions, \( \frac{\partial B^2}{\partial x} \) and \( \frac{\partial B^2}{\partial z} \) (Chapra & Canale 2002). Since the magnetic energy density of the flux tube is greatest at the center of the tube (Figure 6), the change in magnetic energy density with respect to the x- and z-directions should be zero where no magnetic energy density exists and where the flux tube reaches its maximum energy density (Figure 7). To eliminate all of the data points where no magnetic energy density exists, we multiply the array by a 128^3*300 Boolean operator whose value is zero at all points where the magnetic energy density equals zero, and is one at all points where the magnetic energy density does not equal zero. Next, we multiply the array by a Boolean operator that sets all points in the array equal to 0 whose derivative is not equal to zero, and 1 otherwise. In this fashion, we determine where the gradient of the numerical data set is zero.

Theoretically, all that should remain is an array whose value is zero at all data points other than at the center of the flux tube, so we can determine the x- and z-coordinates of the center of the flux tube within y cross-sections. According to Figure 8, the center of
the flux tube exists at \( x = 52 \) and \( z = 148 \). See *Program One: Numerical Differentiation* in the appendix for a template of the program we use to calculate the gradient.

Next, we apply the same method to a data array after convective forces have contorted the flux tube. The method yields an array with many non-zero coordinates, not just one (Figure 9), because once convection has begun to pull bits of magnetic energy away from the flux tube, many little concentrated regions of magnetic energy have regions where \( \frac{\partial B}{\partial x} = 0 \) and \( \frac{\partial B}{\partial z} = 0 \). We decrease the tolerance of the Boolean operator that eliminates zero-magnetic energy density points by making it assign zeroes to points where the magnetic energy density does not reach an arbitrary threshold value. However, we are introducing limits on what properties a coherent flux tube can have, and we have no basis for selecting the limits chosen. We could set the limit to include only points that have magnetic energy density that is a specific fraction of the maximum energy density within each \( x \)-\( z \) cross-section, which would eliminate many more of the points that are probably not in the center of the magnetic flux tube. But we cannot even be sure that the center of the flux tube would have magnetic energy density exceeding that of some regions of strong convective down-flow. So we try a different approach.

*Second Model: Statistical Analysis*

An effective method for describing tube coherence might be to determine the most probable location of the center of a flux tube within a cross-section of the data set and to calculate the sum of the distance of pieces of magnetic energy density from the center of the tube. We calculate the mean magnetic energy density within the array for each of the 128 \( x \)-\( z \) plane cross-sections. The center of mass \( \bar{X} \) of a one-dimensional body can be calculated with equation 4.
where \( m_i \) is the mass of the object in position \( x_i \), and \( n \) is the total number of positions that the particle can have. Expanding equation 4, we calculate the mean location \((\bar{X}, \bar{Z})\) of the magnetic energy density within an x-z plane:

\[
(\bar{X}_j, \bar{Z}_j) = \left( \frac{\sum_{i=0}^{127} \sum_{k=0}^{299} m_{i,j,k} i}{\sum_{i=0}^{127} \sum_{k=0}^{299} m_{i,j,k}}, \frac{\sum_{i=0}^{127} \sum_{k=0}^{299} m_{i,j,k} k}{\sum_{i=0}^{127} \sum_{k=0}^{299} m_{i,j,k}} \right)
\]  
(Eq. 5)

where \( m_{i,j,k} \) is the magnetic energy density in position \((i, j, k)\), \( i \) is the location of a point in plane \( j \) in the x-direction, \( k \) is the location of a point in plane \( j \) in the z-direction, and \( j \) is the x-z plane for which the mean magnetic energy density is calculated. We conduct this calculation for every x-z cross-section, \( j \), or 128 times. Figure 10 shows an example of how well this method traces out the location of a flux tube, and Figure 11 demonstrates how a flux tube starts at \( z = 150 \) and floats upward toward \( z = 0 \), and then is pumped back downward by convective forces.

We can also calculate the variance of the magnetic energy density, much like a typical calculation of standard deviation:

\[
s^2 = \frac{1}{n} \sum_{i=1}^{n} m_i (x_i - \bar{X})^2
\]  
(Eq. 6)

where \( m_i \) is the probability that the particle is in position \( x_i \), \( n \) is the total number of positions that the particle can have, and \( \bar{X} \) is the mean location of the particle. We calculate the variance of magnetic energy density within each x-z plane:
where \( m_{i,j,k} \) is the energy density in position \((i, j, k)\), \( i \) is the location of a point in plane \( j \) in the \( x \)-direction, \( \bar{I} \) is the mean location of the flux tube on the \( x \)-axis, \( k \) is the location of a point in plane \( j \) in the \( z \)-direction, and \( \bar{K} \) is the mean location of flux tube on the \( z \)-axis.

Next, we take the square root of the variances to get the standard deviations of magnetic energy density within each plane, and we sum the standard deviations from each plane:

\[
S_j^2 = \frac{\sum_{i=0}^{127} \sum_{k=0}^{299} \left[ m_{i,j,k} (i - \bar{I}_j)^2 + m_{i,j,k} (k - \bar{K}_j)^2 \right]}{\sum_{i=0}^{127} \sum_{k=0}^{299} m_{i,j,k}}
\]

(Eq. 7)

The sum of the square root of the variances, which henceforth we call the \( s \) value, is an effective indicator of the spread of the magnetic energy density within the domain. It accounts for the density and location of all points in the domain equally. Figure 12 shows how the sum of the standard deviations changes with each time step for Case 125. As time progresses, the \( s \) value increases and eventually levels out. See Program Two: Statistical Analysis in the appendix for a template of the program we use to calculate the mean locations of the tubes and their standard deviations.

**Analysis**

Our next step is to compare \( s \) with the sets to see if the calculations are in accord with what we observe. We might think that the flux tubes in all the different cases should lose coherence at the same standard deviation sum. Figure 13 shows this is not the case. Flux tubes in some cases lose coherence at \( s = 50 \), while in other cases, the flux tubes lose coherence at much earlier \( s \) values. In actuality, we should expect significant
differences in $s$ for cases that are significantly different. For example, in Case 140 where $\alpha$ is 120, $s$ increases quickly, and $s \approx 50$ by $t = 25,000$, but the tubes still appear to be coherent. However, in Case 108, $\alpha$ is 0.024 and 24, respectively, and $s \approx 50$ by $t = 55000$. But the tubes in these cases are incoherent long before then. The various parameters that determine the strength of the flux tube and the vigor of convection also play a part in determining what values of $s$ would predict flux tube incoherence.

**Magnetic Field Strength $\alpha$**

Referring back to Table 1 on page 10, we choose sets whose comparison would best give us an understanding of the effects of changing different parameters. Cases 108, 111, 123, and 124 have the same parameters excluding $\alpha$. Figure 14 is a plot of the $s$ values of these cases as a function of time. In cases with smaller $\alpha$ values (weaker magnetic field strength) the $s$ values increase at a consistent rate, while in cases with larger $\alpha$ values the $s$ values increase rapidly in the first time steps but tend to level out as time increases. A possible explanation for the initially rapid increase of the $s$ value for cases with large $\alpha$ values is that magnetic flux tubes with strong magnetic fields tend to rise more quickly than magnetic flux tubes with weak magnetic fields, and so quickly rising flux tubes leave residual magnetic energy as they rise. Since cases with large $\alpha$ values have greater magnetic field strengths, more magnetic energy will diffuse, contributing to a rapidly increasing $s$ value (Figure 15).

Although the stronger tubes exhibit a rapid increase in their $s$ values, they remain coherent for longer periods of time. The magnetic energy in Case 108 with $\alpha = 0.024$ rapidly diffuses and loses tube coherence by $t = 15,000$. In Case 111 with $\alpha = 24$, the tube remains coherent until $t = 35,000$; and in Case 123 with $\alpha = 48$ the tube may lose
coherence at the end of the simulation, around $t = 50,000$. However, in Case 124 with $\alpha = 96$ the tube appears to remain coherent throughout the duration of the simulation (Figure 16). To see if there is a correlation between $\alpha$ and $s$, we plot the $s$ value of the array at which point the flux tube loses coherence versus the $\alpha$ value used (Figure 17). Sets with higher $\alpha$ values tend to lose coherence at larger $s$ values. See Table 2 on page 19 for a summary of these results.

*Magnetic Diffusivity $\zeta C_k$*

Cases 111 and 125 can be compared to describe how the diffusivity constant affects flux tube coherence because excluding $\zeta$, their parameters are the same. Case 111 has $\zeta = 0.001$, and Case 125 has $\zeta = 0.0001$, which implies that magnetic energy diffuses ten times faster in Case 111 than in Case 125. As Figure 4 shows, pockets of weak magnetic energy that are typically distributed throughout the arrays are less apparent in Case 111 than in Case 125. Since the diffusivity tends to spread and smooth out concentrated regions of magnetic energy, the energy in weak magnetic structures will quickly diffuse; fine magnetic structures rapidly disappear, while strong magnetic structures (such as flux tubes) slowly weaken and eventually lose coherence.

Because magnetic energy diffuses much faster in Case 111, we would expect the magnetic energy to spread more quickly, so the $s$ values would increase faster for Case 111. According to Figure 18, the $s$ values increase much more rapidly for the case with a smaller magnetic diffusivity, Case 125. However, the flux tube in Case 111 loses coherence at about $t = 35,000$ where $s = 32.7$, whereas the flux tube in Case 125 appears to lose coherence at about $t = 65,000$ where $s = 47.0$ (Figure 19), so even though the $s$
value increases more rapidly, the tube with a smaller magnetic diffusivity loses coherence sooner. See Table 3 on page 19 for a summary of these results.

*Viscosity* $P_r C_k$

To gain an understanding of how viscosity impacts the length flux tubes survive, we can compare Case 124 to Case 136. Though Cases 124 and 136 differ in both their $\zeta$ values and their $C_k$ values, the product of $\zeta$ and $C_k$ is the nearly the same for Cases 124 and 136, demonstrating that magnetic diffusivity is nearly the same for Case 124 as it is for 136. $C_k$ impacts the thermal diffusivity, magnetic diffusivity, and viscosity, but the impacts of changing thermal diffusivity are negligible, and $\zeta C_k$ is nearly equal in both cases. Since $P_r C_k$ is different in both cases, we may compare these two cases to study the impacts of different viscosities on flux tube survival length. Case 124 has $P_r C_k = 0.04899$ while Case 136 has $P_r C_k = 0.02499$, so the viscosity is greater in Case 124 than in Case 136.

According to Figure 20, the $s$ value for Case 136 increases much more rapidly than in Case 124, since the solar fluid is moving at a more rapid rate and mixing the magnetic energy density more violently. The tube in Case 136 appears to lose coherence at about $t = 55,000$ where $s = 47.6$. The tube in Case 124 appears to remain coherent throughout the duration of the simulation, which lasts to $t = 125,000$ where $s = 47.0$ (Figure 21). Although we cannot draw accurate generalizations on just two sets, it would appear that tubes in sets with larger $P_r$ values tend to remain coherent for longer periods of time, but the tubes in different cases might lose coherence at similar $s$ values. See Table 4 on page 19 for a summary of these results.
**Thermal Diffusivity C_k**

Case 140 and 142 differ only in their values of $C_k$, implying that they differ in the rate of thermal diffusivity, magnetic diffusivity, and viscosity. Case 140 has $C_k = 0.2499$ while Case 142 has $C_k = 0.06928$, so the solar fluid in Case 140 is more viscous and magnetic energy diffuses at a faster rate than in Case 142. Because Case 140 is more viscous, we would expect the $s$ value to increase at a slower rate. However, since the magnetic diffusivity is greater in Case 140, we would expect the $s$ value to increase at a faster rate. According to Figure 22, the $s$ value increases at a faster rate for Case 142, indicating that in these simulations viscosity plays a more active role.

The flux tube in Case 140 loses coherence at about $t = 65,000$ where $s = 54.8$ and the flux tube in Case 142 loses coherence at about $t = 20,000$ where $s = 52.4$ (Figure 23). Although the tube in Case 140 lasts substantially longer, the $s$ values that correspond to loss of coherence are very close. See Table 5 on the following page for a summary of these results.
Results

<table>
<thead>
<tr>
<th></th>
<th>Tube coherence loss at t</th>
<th>s value corresponding to loss of coherence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 108</td>
<td>0.024</td>
<td>15,000</td>
</tr>
<tr>
<td>Case 111</td>
<td>24.</td>
<td>35,000</td>
</tr>
<tr>
<td>Case 123</td>
<td>48.</td>
<td>50,000+</td>
</tr>
<tr>
<td>Case 124</td>
<td>96.</td>
<td>125,000+</td>
</tr>
</tbody>
</table>

Table 2: A table containing $\alpha$ values for four cases, the length of time the tube in each case remains coherent, and the $s$ value corresponding to coherence loss. For these cases, all other parameters are equal.

<table>
<thead>
<tr>
<th></th>
<th>Tube coherence loss at t</th>
<th>s value corresponding to loss of coherence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 111</td>
<td>0.001</td>
<td>35,000</td>
</tr>
<tr>
<td>Case 125</td>
<td>0.0001</td>
<td>65,000</td>
</tr>
</tbody>
</table>

Table 3: A table containing $\zeta$ values for two cases, the length of time the tube in each case remains coherent, and the $s$ value corresponding to coherence loss. For these cases, all other parameters are equal.

<table>
<thead>
<tr>
<th></th>
<th>Tube coherence loss at t</th>
<th>s value corresponding to loss of coherence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 124</td>
<td>0.04899</td>
<td>125,000+</td>
</tr>
<tr>
<td>Case 136</td>
<td>0.02499</td>
<td>35,000</td>
</tr>
</tbody>
</table>

Table 4: A table containing $P_r C_k$ values for two cases, the length of time the tube in each case remains coherent, and the $s$ value corresponding to coherence loss. For these cases, all other parameters are equal.

<table>
<thead>
<tr>
<th></th>
<th>Tube coherence loss at t</th>
<th>s value corresponding to loss of coherence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 140</td>
<td>0.2499</td>
<td>65,000</td>
</tr>
<tr>
<td>Case 142</td>
<td>0.06928</td>
<td>20,000</td>
</tr>
</tbody>
</table>

Table 5: A table containing $C_k$ values for two cases, the length of time the tube in each case remains coherent, and the $s$ value corresponding to coherence loss. For these cases, all other parameters are equal.
Conclusion

Observations

Based on qualitative and quantitative observations, we conclude that flux tubes with large $\alpha$ values tend to last longer than tubes with small $\alpha$ values. Tubes in highly viscous environments tend to survive longer than tubes in environments with low viscosity. Tubes with a small magnetic diffusivity tend to last for a longer period of time than tubes with a large magnetic diffusivity.

If the parameters of these simulations are known, the $s$ values can be used to estimate the time at which flux tubes lose coherence. For cases differing in only the $\alpha$ values, the relationship between $s$ and $\alpha$ may be modeled by a monotonic function, possibly logarithmic. We do not have enough data sets to find a correlation between $\zeta$ values and $s$ values. Viscosity does not appear to significantly impact the $s$ value at which the tubes lose coherence.

One peculiarity that arises from the results of our comparison of Case 140 to 142 is that changing $C_k$ does not appear to impact the $s$ value at which the flux tubes lose coherence. But if changing the magnetic diffusivity does seem to impact the $s$ value where the tubes lose coherence, while changing the viscosity does not, then it would be expected that changing $C_k$ would impact the $s$ value, since $C_k$ impacts both the magnetic diffusivity and the viscosity. Possibly the factor by which $P_r C_k$ from Case 124 to Case 136 differs may not be large enough to reveal that the $s$ value at which flux tubes lose coherence does change. If we had more sets to analyze, we could draw more accurate conclusions on how the parameters and the $s$ values could be correlated.
**Strengths, Weaknesses and Future Work**

A major weakness in this paper is that it does not compare and analyze enough sets to determine solid correlations between the $s$ values and the parameters. If more data sets could be analyzed, it might be possible to find stronger correlations. Also, for many of the data sets, the $s$ value tends to level out as time increases, so sometimes the $s$ value is greater for times previous to when it lost coherence. For example, in Case 142, the flux tube is said to lose coherence at $t = 20,000$ where $s = 52.4$, but at $t = 15,000$, the flux tube is still coherent, and the array's $s$ value is 53.5. For most cases this does not happen, but it indicates that improvements could be made on the statistical model.

The statistical model calculates the variance only within $x$-$z$ planes of the domain, but the flux tube does not necessarily have to be perpendicular to that plane at any given time (Figure 24). The tube stretches so to a position almost parallel to the $x$-$z$ plane at one point. Because of the great spread of the magnetic energy density within this plane, the $s$ value is very large for this cross-section.

One method to consider would be to calculate the change in flux tube location (based on the mean location) within each $x$-$z$ plane to determine the direction that the tube is twisting, and then to determine the spread of the magnetic energy density within a plane that is perpendicular to the flux tube and intersects with the flux tube at the point where its direction is determined. However, this method would give more weight to certain points in the domain and no weight to others. In addition, such a plane might intersect with the flux tube at several different points, not just the point that we want it to, which would erroneously increase the value of the standard deviation that we calculate. For the most part, however, numerical and observational analyses tend to agree.
Works Cited

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Figure 1: A butterfly diagram indicating that average sunspot incidence (as expressed by the longitudinally averaged magnetic field) migrates toward the equator of the sun during the 11-year cycle. Note that with the beginning of the 1977 11-year cycle, the magnetic fields of the westernmost sunspots in the southern hemisphere point radially inward, while in the next cycle, their magnetic fields point radially outward (Hathaway 2005).
Figure 2: The sun's initially poloidal magnetic field is wrapped around the sun by differential rotation, producing toroidal magnetic fields pointing in opposite directions within each hemisphere (Hathaway 2005).
Figure 3: A computer-generated flux tube before convection has acted on it. The y-axis runs parallel to the sun’s latitude (parallel to the flux tube) and the x-axis runs parallel the sun’s longitudes (orthogonal to the flux tube). The z-axis points radially inward, so $z = 300$ is closer to the center of the sun than $z = 0$. 
Figure 4: A volume rendering of the magnetic energy density for (a) Case 108 step 25,000 with $\alpha = 0.024$ and (b) Case 111 step 25,000 with $\alpha = 24$. By step 25,000 Case 108 does not have a coherent tube-like structure, whereas Case 111 still has an apparent tube.
Figure 5: A volume rendering of the magnetic energy density for (a) Case 111 step 25,000 with $\zeta = 0.001$ and (b) Case 126 step 25,000 with $\zeta = 0.0001$. The magnetic energy density has diffused much more in Case 111 than in Case 125.
Figure 6: A surface plot of the magnetic energy density for Case 142 step 0 at y = 64. We assume that the center of the tube exists where the tube reaches a maximum energy density.
Figure 7: The change in magnetic energy with respect to (a) the x-axis and (b) the z-axis. For (a) the black region is an area where $\frac{\partial B^2}{\partial x} > 0$ and the white region is an area where $\frac{\partial B^2}{\partial x} < 0$. For (b) the black region is an area where $\frac{\partial B^2}{\partial z} > 0$ and the white region is an area where $\frac{\partial B^2}{\partial z} < 0$. Between the increasing and decreasing regions is a region where both $\frac{\partial B^2}{\partial x} = 0$ and $\frac{\partial B^2}{\partial z} = 0$. This is the center of the flux tube.
Figure 8: (a) is a surface plot of the magnetic energy density for Case 142 step 0 at $y = 64$. The center of the flux tube is where the tube has a maximum value. (b) is a surface plot that is non-zero where the gradient of the magnetic energy density is zero. According to plot (b), the flux tube's center is at $x = 52$ and $z = 148$. 
Figure 9: (a) The surface plot for the magnetic energy density in Case 140 step 25,000 at \( y = 64 \) shows a prominent peak where the flux tube exists. However, when the gradient algorithm is applied (b), it is not clear where the magnetic flux tube is, because there are many points where the changes in magnetic energy with respect to the \( x \)- and \( z \)-directions are zero.
Figure 10: (a) is an aerial view of the volume rendering of the magnetic energy density for Case 140 step 10,000, and (b) is the calculated mean magnetic energy density for given x- and y-coordinates. I use the mean magnetic energy density to describe the mean location of the flux tube.
Figure 11: A surface plot of the mean magnetic energy density in the z-axis for Case 140 steps 0 (0) through 50,000 (10). A step 0, the tube is at $z = 150$. Because of buoyancy the tube rises to about $z = 40$, after which time regions of strong convective downflow pump the tube back down.
Figure 12: As time progresses the s value tends to increase, eventually leveling out.
Figure 13: Magnetic energy density plots for (a) Case 108 step 55,000 where \( s = 50.058 \) and for (b) Case 142 time step 15,000 where \( s = 53.465 \). Although neither cases appear to have coherent tubes at their respective time steps, the appearance of Case 108 step 55,000 suggests that the tube lost coherence long before step 55,000, whereas it would appear as if the tube in Case 142 just recently lost coherence.
Cases with different alpha values

Figure 14: A plot of $s$ versus time for four different cases differing in $\alpha$. For Case 108, 111, 123, and 124, $\alpha = 0.024, 24, 48$, and 96, respectively. Initially, the $s$ value in cases with higher $\alpha$ values (a greater magnetic energy) increase faster than those with lower $\alpha$ values. But the $s$ values level out sooner in the cases with higher $\alpha$ values.
Figure 15: Magnetic energy density plots for (a) Case 111 step 15,000 where $\alpha = 24$, and (b) case 124 step 15,000 where $\alpha = 96$. The flux tube in case 124 has already risen to the surface of the convective zone ($z = 0$). Since the magnetic energy density in Case 124 is greater, it is possible that diffusion has impacted the spread of magnetic energy density more in Case 124.
Figure 16: Magnetic energy density plots of (a) Case 108, $t = 15,000$, $a = 0.024$; (b) Case 111, $t = 35,000$, $a = 24$; (c) Case 123, $t = 50,000$, $a = 48$; and (d) Case 124, $t = 125,000$, $a = 96$. The tubes in (a) and (b) are deemed incoherent, and the tubes in (c) and (d) are nearly incoherent.
Figure 17: A plot of the $s$ value for which the tube is deemed incoherent versus the $\alpha$ value for Cases 108, 111, 123, and 124. It appears as if the $s$ value that corresponds to tube incoherence approaches 50 for cases with large $\alpha$ values and other parameters equal to those of the four cases.
The Effect of Magnetic Diffusivity on $s$

Figure 18: A plot of $s$ versus time for Case 111 where $\zeta = 0.001$ and for Case 125 where $\zeta = 0.0001$. The $s$ values increase more rapidly for Case 111, the case with a smaller magnetic diffusivity.
Figure 19: Magnetic energy density plots of (a) Case 111, $t = 35,000, \zeta = 0.001$, and (b) Case 125, $t = 65,000, \zeta = 0.0001$. At these times, both tubes are regarded as incoherent.
The Effect of Viscosity on $s$

Figure 20: A plot of $s$ versus time for Case 124 where $P_1 C_k = 0.04899$ and for Case 136 where $P_1 C_k = 0.02499$. Since the solar fluid in Case 124 is more viscous, the velocity of the solar fluid is slower. So magnetic energy density is spread out at a slower rate.
Figure 21: Magnetic energy density plots of (a) Case 124, $t = 125,000$, $P_r C_k = 0.04899$, and (b) Case 136, $t = 55,000$, $P_r C_k = 0.02499$. The tube in Case 136 is in a less viscous environment. At these times, the tube in Case 124 is nearly incoherent and the tube in Case 136 is regarded as incoherent.
Figure 22: A plot of $s$ versus time for Case 140 where $C_k = 0.2499$ and for Case 142 where $C_k = 0.06928$. Magnetic diffusivity occurs at a greater rate in Case 140, but the fluid in Case 140 is more viscous, so the $s$ value increases faster in Case 142.
Figure 23: Magnetic energy density plots of (a) Case 140, $t = 65,000$, $C_k = 0.2499$, and (b) Case 142, $t = 20,000$, $C_k = 0.06928$. The tube in Case 140 is in a less viscous environment but has a greater magnetic diffusivity. At these times, both tubes are considered incoherent.
Figure 24: A volume plot of the magnetic energy density for Case 125, $t = 70,000$. A region of strong convective downflow has pulled a portion of the tube to the base of the convective zone. The standard deviation is very large within the x-z plane where the tube is parallel to the z-axis.
Appendix

Program One: Numerical Differentiation

for i=long(#####),long(#####),long(5000) do begin
  cd,strcompress('S:kcline\Research\Case_###\'+string(i/10000)+string(i/10000 mod 10)+string(i/1000 mod 10)+string(i/100 mod 10)+string(i/10 mod 10),/remove_all)
  bx=read_binary('Bx',data_type=4,data_dims=[128,128,300],endian='big')
  by=read_binary('By',data_type=4,data_dims=[128,128,300],endian='big')
  bz=read_binary('Bz',data_type=4,data_dims=[128,128,300],endian='big')
  bb0=bx^2+by^2+bz^2
  dbb0dx=(shift(bb0,2,0,0)+8*shift(bb0,1,0,0)-8*shift(bb0,-1,0,0)+shift(bb0,-2,0,0))/12
  dbb0dz=(shift(bb0,0,0,2)+8*shift(bb0,0,0,1)-8*shift(bb0,0,0,-1)+shift(bb0,0,0,-2))/12
  grad0=(abs(dbb0dx lt 0.0001))*abs(dbb0dz lt 0.0001))*bb0
endif
end

This program is written for IDL
Program Two: Statistical Analysis

steps==
bb=fltarr(128,128,300,steps)
mx=fltarr(128)
mz=fltarr(128)
wbb=fltarr(128,128,300)
s=fltarr(steps)
sarray=fltarr(steps,128)
mxarray=fltarr(steps,128)
mzarray=fltarr(steps,128)
for p=long(#####),long(#####),long(5000) do begin
print,p
cd,strcompress('S:\kcline\Research\Case_###\'+string(p/100000)+string(p/10000 mod 10)+string(p/1000 mod 10)+string(p/10 mod 10)+string(p mod 10),/remove_all)

bx=read_binary('Bx',data_type=4,data_dims=[128,128,300],endian='big')
by=read_binary('By',data_type=4,data_dims=[128,128,300],endian='big')

bz=read_binary('Bz',data_type=4,data_dims=[128,128,300],endian='big')
bb0=bx^2+by^2+bz^2
bb(*,*,*,p/5000)=bb0
s2=fltarr(128)
s0=fltarr(128)
for j=0,127 do begin
mx0=0
mz0=0
wbb(*,j,*)=bb0(*,j,*)/total(bb0(*,j,*)
for i=0,127 do for k=0,299 do mx0=mx0+wbb(i,j,k)*i
for i=0,127 do for k=0,299 do mz0=mz0+wbb(i,j,k)*k
for i=0,127 do for k=0,299 do s2(j) = s2(j) + wbb(i,j,k)*(i-mx0)^2+wbb(i,j,k)*(k-mz0)^2
s0(j)=sqrt(s2(j))
mx(j)=mx0
mz(j)=mz0
endfor
s0=sqrt(s2)
s(p/5000)=mean(s0)
sarray(p/5000,*)=s0
mxarray(p/5000,*)=mx
mzarray(p/5000,*)=mz
endfor
end

This program is written for IDL