Spring 1971

Theory Of Monopolistic Competition

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Theory of Monopolistic Competition

By Michael Wing-Wah Tam

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14 April 1971
(Date)
This thesis for honors recognition has been approved for the Department of Business and Economics.

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In the history of American economy, the complex mixture, confusion and separation of the ideas of competition and monopoly are always found. Although the two forces are two basic extremes according to the economic theory, yet they are interwoven, with a variety of design, throughout the price system, each bearing to it a strong resemblance.

In this paper, I have developed a special technique, apart from the traditional approach, to attack the problem of monopolistic competition with the precision of mathematical methods.

The first step I will take, is to give a clear definition of the two fundamental forces of competition and monopoly, in the formulation of a theory of prices, and an examination of each in isolation.

The second step must be a synthesis of the two. This brings back to the price theories of the two extreme channels, without the recognition of the middle course.

Thirdly, I will try to evaluate the value under pure competition by distinguishing equilibrium from the equation of supply and demand; the individual sellers under pure competition and their cost curves and scales of production.

Fourthly, I will attempt to analyze the present structure of duopoly and oligopoly. By doing this, I have managed to investigate and discover the mutual independence among the sellers; which finally lead to the effect of uncertainty.
I. Monopoly and Perfect Competitor

Since no competition exists in a monopolistic market, there is no distinction between the industry and the firm. The industry is the firm, and the firm is the industry. Though a monopolist's demand curve is always steeper in slope, it generally possesses the similar properties as the demand curve for a perfectly competitive market. They are both negatively sloped; and they are both aggregates of the demand curves of individual demand curves. Thus, the quantity of his sales is a single valued function of the price which he charges:

\[ q = f(p) \quad (1) \]

This demand curve has a unique inverse. Hence, price may also be expressed as a single-valued function of quantity:

\[ p = f(q) \quad (2) \]

where \( \frac{dp}{dq} < 0 \) and \( \frac{dq}{dp} < 0 \) slope of demand curve is \( -ve \).

From these two equations, we can already discover a major difference between a monopolist and a perfect competitor. A monopolist can use both equation 1 and 2; while a perfect competitor can only use equation 1. In other words, a monopolist can decrease his price as he increases his sales; a perfect competitor accepts price as a parameter and he can only maximize profit by varying his output level. The monopolist has both variables to work with, while the perfect competitor has only one —— \( q \) (output).
The monopolist’s total revenue (TR) is simply the product of price and quantity sold:

\[ TR = pq \]  \hspace{1cm} (3)

His marginal revenue (MR) is the derivative of his total revenue with respect to his output. Therefore,

\[ MR = \frac{d(TR)}{dq} = p \frac{dq}{dq} + q \frac{dp}{dq} = p + q \frac{dp}{dq}. \] \hspace{1cm} (4)

For the monopolist,

Since \( \frac{dL}{dq} < 0 \), \( q \frac{dp}{dq} < 0 \); \[ \Rightarrow \] MR < p.

For the perfect competitor,

\[ \frac{dp}{dq} = 0; \quad q \frac{dL}{dq} = 0 \Rightarrow MR = p. \]

The monopolist's marginal revenue equals price less the product of quantity and the rate of change in price with respect to quantity. If the perfect competitor increases his sales by one unit, his total revenue will increase by the market value of the additional unit. The monopolist, however, must decrease the price he receives for every unit in order to sell an additional unit.

Let \( p = a - bq \) \hspace{1cm} (5)

\[ TR = pq = aq - bq^2 \] \hspace{1cm} (6)

\[ MR = \frac{d(TR)}{dq} = a - 2bq \] \hspace{1cm} (7)

\[ \Rightarrow M_{MR} = 2M_{DD} \] \hspace{1cm} (8)
The slope of the marginal revenue curve is generally twice the slope of the demand curve. Demand is monotonically decreasing, and marginal revenue is less than price for every output greater than zero.

For every \( q > 0 \); \( MR < p \).

This result is applicable to demand curves which are not linear. In general

\[
\int_0^q \left( p + \frac{dp}{dq} \right) dq = pq - q
\]

since the integration constant always equals zero. Total revenue is always given by the area lying under the MR curve.

The elasticity of demand \( \varepsilon \) at a point on a demand curve is defined as the absolute value of the rate of percentage change of output divided by the rate of percentage change of price:

\[
\varepsilon = \frac{d(\log q)}{d(\log p)} = \frac{p dq}{q dp}
\]

MR as given by (4) can be expressed in terms of price and demand elasticity:

\[
MR = p \left( 1 + \frac{q \cdot dp}{p \cdot dq} \right) = p \left( 1 - \frac{1}{\varepsilon} \right)
\]

MR is positive if \( \varepsilon < 1 \), zero if \( \varepsilon = 1 \), and negative if \( \varepsilon > 1 \). The difference between MR and price decreases as demand elasticity increases, and MR approaches price as demand elasticity approaches infinity.
Since \( \frac{dp}{dq} = -b \) is a constant, the distance between the two curves \( \left( \frac{dq}{dp} = bq \right) \) is a linear function of output. Total revenue for the price-quantity combination \((p_0, q_0)\) equals the area of the rectangle \(0p_0Tq_0\).

The area \( OASq_0 \) which lies under the MR curve also equals total revenue:

\[
\int_0^q (a - 2bq) \, dq = aq - bq^2 = R \tag{9}
\]

This result is applicable to demand curves which are not linear. In general

\[
\int_0^q \left( p + \frac{dp}{dq} \right) \, dq = pq = R \tag{10}
\]

since the integration constant always equals zero. Total revenue is always given by the area lying under the MR curve.

The elasticity of demand \(e\) at a point on a demand curve is defined as the absolute value of the rate of percentage change of output divided by the rate of percentage change of price:

\[
e = -\frac{d(\log q)}{d(\log p)} = -\frac{p \, dq}{q \, dp} \tag{11}
\]

MR as given by (4) can be expressed in terms of price and demand elasticity:

\[
MR = p \left( 1 + \frac{q \, dp}{p \, dq} \right) = p \left( 1 - \frac{1}{e} \right) \tag{12}
\]

MR is positive if \(e > 1\), zero if \(e = 1\), and negative if \(e < 1\). The difference between MR and price decreases as demand elasticity increases, and MR approaches price as demand elasticity approaches infinity.
A parabolic total revenue curve which corresponds to the linear demand curve of Fig. 3 is presented in Fig. 4. The first derivative of total revenue (MR) is monotonically decreasing and reaches zero at the output level \( q_0 \). Total revenue is increasing and \( e > 1 \) for \( q < q_0 \), is at a maximum and \( e = 1 \) for \( q = q_0 \), and is declining and \( e < 1 \) for \( q > q_0 \).

The monopolist's total revenue and total cost can both be expressed as functions of output:

\[
R = R(q) \quad C = C(q)
\]

His profit is the difference between his total revenue and total cost:

\[
\Pi = R(q) - C(q)
\]

To maximize profit set the derivative of (13) with respect to \( q \) equal to zero:

\[
R'(q) - C'(q) = 0
\]

or \( R'(q) = C'(q) \) (14)

\( MR \) must equal \( MC \) for profit maximization. The monopolist can increase his profit by expanding (or contracting) his output, as long as the addition to his revenue (\( MR \)) exceeds (or is less than) the addition to his cost (\( MC \)).

The second-order condition for profit maximization requires that

\[
R''(q) - C''(q) < 0
\]

or adding \( C''(q) \) to both sides of the inequality,

\[
R''(q) < C''(q)
\]

(15)
The rate of increase of MR must be less than the rate of increase of MC. The second-order condition is a fortiori satisfied if MR is decreasing and MC increasing, as is generally assumed. If MC is decreasing, (15) requires that MR be decreasing at a more rapid rate. If both conditions for profit maximization are satisfied for more than one output level, the one which yields the greatest profit can be selected by inspection.

The first-order condition can be satisfied in each of the three cases presented Fig. 5. The equalization of MR and MC for (a) determines a quantity of \( q_o \) and a price of \( p_o \). The monopolist can set the price \( p_o \) and allow the consumers to purchase \( q_o \), or he can offer \( q_o \) for sale and allow the consumers to determine a price of \( p_o \). The second-order condition requires that the algebraic value of the slope of the MC curve exceed that of the MR curve, i.e., the MC curve must cut the MR curve from below. This condition is satisfied at the intersection points in (a) and (b). There is no point of maximum profit in (c) since the MC curve cuts the MR curve from above at their only point of intersection. The first-order condition can be satisfied, but the second-order condition cannot.

If a monopolist followed the rule of a perfect competitor and equated MC to price, he would produce a greater output and charge a lower price. This is obvious by Fig. 5(a). The coordinates of the intersection point of the MC and demand curves give a price less than \( p_o \) and a quantity greater than \( q_o \).
Suppose a monopolist has a linear demand curve:

\[ p = 120 - 5q \]

\[ R = pq = 120q - 5q^2 \]

\[ MR = 120 - 10q \quad (16) \]

and produces at a constant MC of $20. His total cost is a linear function of his output.

\[ C = 50 + 20q \quad (17) \]

\[ MC = 20 \]

Therefore, the monopolist’s profit of $450 would be, in this case, reduced to a $50 loss.

\[ \pi = R(q) - C(q) \]

\[ = (120q - 5q^2) - (50 + 20q) \]

When profit is maximized, \( MR = MC \)

\[ 120 - 10q = 20 \]

\[ -10q = -100 \]

\[ q = 10 \]

\[ p = 120 - 5(10) = 70 \]

\[ \pi = 100q - 5q^2 - 50 \quad (18) \]

\[ = 1000 - 5000 - 50 \]

\[ = 450 \]

The second order condition is satisfied, since the rate of change of MC (zero) exceeds the rate of change of MR (-10). If the monopolist were to follow the rule of the competitor and set MC = p
120 - 5q = 20

In this section, we are particularly interested in finding the relationship between corporate profit tax and the output level of a monopolist. As we can intuitively see, the corporate profit tax will automatically reduce the profit after taxes of a profit-maximizing monopolist. Such a tax, however, can never affect the price-quantity combination.

Thus, the monopolist's profit of $450 would be, in this case, reduced to a $50 loss.

\[ \pi = \Pi - \tau(q) - c(q) - t \]

where \( \pi \) = profit after tax
\( \Pi \) = the amount of the lump-sum tax paid by the monopolist
\( \tau(q) \) = the amount of the lump-sum tax paid by the monopolist
\( c(q) \) = the amount of the lump-sum tax paid by the monopolist

For profit maximization, set the derivative equal to zero.

\[ \frac{d\pi}{dq} = \frac{d\Pi}{dq} - \frac{dc}{dq} - \frac{d\tau}{dq} = 0 \]

Therefore, \( \frac{d\Pi}{dq} = \frac{dc}{dq} + \frac{d\tau}{dq} \).

The monopolist cannot avoid a lump-sum tax. It has to be paid no matter what the quantity or value of his sales are. Since the lump-sum tax is a constant in (19), it vanishes upon differentiation. Even though the tax is imposed, the first-order and second-order conditions are
II. Government's Role: Taxation

In this section, I am particularly interested in finding the relationship between corporate profit tax and the output level of a monopolist. We can intuitively see that corporate profit tax will automatically reduce the profit after taxes of a profit-maximizing monopolist. Such a tax, however, can never affect the price-quantity combination. This can be shown by the following equation:

The monopolist's profit

\[ \Pi_1 = R(q) - C(q) - Tx \]  \hspace{1cm} (20)

where \( \Pi_1 \) = profit after tax

\( Tx \) = the amount of the lump-sum tax paid by the monopolist.

For profit maximization, set the derivative equal to zero

\[ \frac{d \Pi_1}{dq} = R'(q) - C'(q) - 0 = 0 \]

\[ \rightarrow R'(q) - C'(q) = 0 \]

Therefore, \( R'(q) = C'(q) \)

The monopolist cannot avoid a lump-sum tax. It has to be paid no matter what the quantity or value of his sales are. Since the lump-sum tax is a constant in (20), it vanishes upon differentiation. Even though the tax is imposed, the first-order and second-order conditions are
both satisfied.¹

For the case of a profit tax (with a marginal rate less than 100%), a specified proportion of the difference between his TR and TC.

Let \( t \) be the percentage rate of tax paid as a profit tax.

\[
\Pi_2 = R(q) - C(q) - t[R(q) - C(q)]
\]

\[= (1 - t) \left[ R(q) - C(q) \right] \]

(21)

Since \( t < 100\% \); \( 0 < t < 1 \).

Setting the derivative equal to zero.

\[
\Pi_2' = \frac{d \Pi_2}{dq} = (1 - t) \left[ R'(q) - C'(q) \right] = 0
\]

Since \((1-t) \neq 0\),

\[R'(q) - C'(q) = 0\]

\[\Rightarrow R'(q) = C'(q)\]

Thirdly, I like to investigate the consequence of a sales tax. Presently, a sales tax is imposed by each individual state on the basis of a specified percentage of total amount of sales.

¹Again, the assumption that all products compete for the limited incomes of consumers prevails here. The prices of all other commodities are assumed constant, as is always the case for the analysis of a single market, and the competition of other commodities for the consumer's income is indicated in the position and characteristic of the monopolist's demand curve.
Hence,
\[ \Pi = R(q) - C(q) - \beta q \]  
\[ \Pi'(q) = R'(q) - C'(q) - \beta = 0 \] (22)

Differentiate (23) once more,

\[ R''(q) = C'(q) + \frac{d\beta}{dq} \] (24)

and

\[ R''(q)dq = C'(q)dq + d\beta \] (25)

Since \( R''(q) - C''(q) < 0 \), \( \frac{dq}{d\beta} < 0 \).

This indicates that as the sales tax increases, the optimum output level would tend to decline. In this case, the imposition of a sales tax results in a smaller quantity sold and a higher price.

Now, let me compare the two types of taxes, namely, the preferability between a lump-sum tax and a percentage sales tax.

Assume that the government imposes a tax of eight dollars per unit upon the monopolist's output:

\[ \Pi = (120q - 5q^2) - (50 + 20)q - 7q \] (26)
\[ \Pi' = \frac{d\Pi}{dq} = (120 - 20 - 7) - 10q = 0 \]

\[ 10q = 9.3 \]

\[ q = 9.3 \] (27)

\[ p = 120 - 5(9.3) \]
\[ = 120 - 46.5 \]
\[ = $73.5 \] (28)

-12-
What does this indicate?

(1) Sales diminish by 0.7 unit. (10 - 9.3)

(2) Price increases by 3.5 dollars.

(3) The government receives the same amount of tax revenue.

(4) Should the firm pay the tax in a lump-sum form, its net profit would be

\[
\text{Net } \Pi = \Pi - \text{lump sum tax}
\]

\[
= \$450 - 7(10)
\]

\[
= \$450 - \$70
\]

\[
= \$380
\]

This simply means that the firm decreases its profit by

\[
(\$380 - \$372.5) = \$7.5
\]

Therefore, in this case, it is proved that a lump-sum tax is preferable to a percentage tax.

Then, we may ask the following question. "Why is the government not consistent with its taxation policy?" or "Why doesn't the government simply adopt a lump-sum tax policy for all industries?"

To answer this question, let us assume that the government imposes a tax of higher than 10%, say 10 dollars per
unit of quantity sold.

\[ \pi = (120q - 5q^2) - (50 + 20q) - 10q \]  \hspace{1cm} (32)

Therefore,

\[ \pi' = \frac{d\pi}{dq} = (120 - 20 - 10) - 10q \]

\[ = 90 - 10q = 0 \]

Therefore,

\[ 10q = 90 \]

\[ q = 9 \]  \hspace{1cm} (33)

\[ p = 120 - 5q \]

\[ = 120 - 5(9) \]

\[ = 75 \]  \hspace{1cm} (34)

\[ \pi = 90(9) - 5(9)^2 - 50 \]

\[ = 810 - 405 - 50 \]

\[ = 355 \]  \hspace{1cm} (35)

Result:

(1) A further cutback in output, down to nine units.

(2) A higher price results, from $73.5 to $75.

(3) The government receives the same amount of tax revenue.

(4) Apparently, profit decreases by some more, from $372.5 to $355.

(5) However, should the firm pay the tax in a lump-sum pattern, its net profit would be

\[ \text{Net } \pi = \$450 - \$10 (10) \]

\[ = \$350. \]
Therefore, its net profit decreases by five dollars. Is this five dollars worth having? It is up to the reader to decide. On an economic philosophical point of view, I feel that by raising the price of five dollars, and cutting back the output by a total of ten per cent in order to increase the profit by five dollars is not only immoral but very detrimental to the economy.

The results are similar if the sales tax is a proportion of the value of sales (total revenue)

\[ \Pi = R(q) - C(q) - XR(q) \]

\[ = (1 - X) R(q) - C(q) \] (36)

\[ \frac{d\Pi}{dq} = (1 - X) R'(q) - C'(q) = 0 \]

\[ (1 - X) R'(q) = C'(q) \]

where \( 0 < X < 1 \)

Taking the second derivative,

\[ (1 - X) R''(q) - R'(q) \frac{dx}{dq} = C''(q) dq \]

Therefore,

\[ \frac{dq}{dx} = \frac{R'(q)}{(1 - X) R(q) - C''(q)} \] (37)

Since the first-order equation that \( MR \) be positive and the second-order condition requires that the denominator of (37) be negative, \( \frac{dq}{dx} < 0 \). The imposition of an ad valorem sales tax also results in a reduced output and an increased price.¹

¹Ad valorem, in this sense, is simply concerned with the value of sales.
The monopolist need not always sell his entire output in a single market for a uniform price. In some situations he is able to sell in two or more distinct markets at different prices and thereby increase his profit. Price discrimination is feasible only if buyers are unable to purchase the product in one market and resell it in another. Otherwise, speculators would buy in a low-price market and resell in a high-price market at a profit, and thereby equalize price in all markets. Personal services are seldom transferable, and their sale frequently provides an opportunity for price discrimination. The resale of such commodities as electricity, gas, and water, which require physical connections between the facilities of the producer and consumer, is extremely difficult, and price discrimination is widely followed in setting utility rates. Price discrimination is often possible in spatially separated markets such as the "home" and "foreign" markets of a monopolist who sells abroad; resale can be prevented by a sufficiently high tariff.

If a monopolist practices price discrimination in two distinct markets, his profit is the difference between his total revenue from both markets and his total cost of production:

\[ \pi(q_1, q_2) = R(q_1) + R(q_2) - C(q_1 + q_2) \]
where $q_1$ and $q_2$ are the quantities which he sells in the two markets, $R_1(q_1)$ and $R_2(q_2)$ are his revenue functions, and $C(q_1 + q_2)$ is his cost function. Setting the partial derivatives of (20) equal to zero,

$$\frac{\partial R}{\partial q_1} = R_1'(q_1) - C'(q_1 + q_2) = 0$$

$$\frac{\partial R}{\partial q_2} = R_2'(q_2) - C'(q_1 + q_2) = 0$$

These imply that

$$R_1'(q_1) = R_2'(q_2) = C'(q_1 + q_2)$$

(39)

The MR in each market must equal the MC of the output as a whole. If the MRs were not equal, the monopolist could increase total revenue without affecting total cost by shifting sales from the low MR market to the high one. The equality of the MRs does not necessarily imply the equality of prices in the two markets. Denoting the prices and the demand elasticities in the two markets by $p_1$, $p_2$, $e_1$, and $e_2$ and utilizing

the equality of the MRs implies

$$p_1\left(1 - \frac{1}{e_1}\right) = p_2\left(1 - \frac{1}{e_2}\right)$$

and

$$p_1 = 1 - \frac{1}{e_2}$$

$$p_2 = 1 - \frac{1}{e_2}$$

(40)

Price will be lower in the market with the greater demand elasticity. The prices will be equal if and only if the demand elasticities are equal.

Second-order conditions require that the principal minors of the relevant Hessian determinant
alternate in sign beginning with the negative sign. Expanding the principal minors,

\[
R''_1 - C'' < 0 \quad (R''_2 - C'')(R''_1 - C'') - (C'')^2 > 0
\]

These imply that \((R''_2 - C'') < 0\). The MR in each market must be increasing less rapidly than the MC for the output as a whole.

Assume that the monopolist whose demand and cost functions are given by (16) and (17) is able to separate his consumers into two distinct markets:

\[
\begin{align*}
p_1 &= 80 - 5q_1 & R_1 &= 80q_1 - 5q_1^2 \\
p_2 &= 180 - 20q_2 & R_2 &= 180q_2 - 20q_2^2 \\
c &= 50 + 20(q_1 + q_2) & (41) \\
c &= 50 + 20q_2 & (42)
\end{align*}
\]

Setting the MR in each market equal to the MC of the output as a whole,

\[
(\frac{R'}{1}) (\frac{C'}{1}) \quad (\frac{R'}{2}) (\frac{C'}{2})
\]

\[
\begin{align*}
80 - 10q_1 &= 20 & 180 - 40q_2 &= 20
\end{align*}
\]

Solving for \(q_1\) and \(q_2\) and substituting into the demand, profit, and elasticity equations,

\[
\begin{align*}
q_1 &= 6 & p_1 &= 50 & \varepsilon_1 &= 1.67 \\
q_2 &= 4 & p_2 &= 100 & \varepsilon_2 &= 1.25
\end{align*}
\]

Second-order conditions are satisfied:

\[
\begin{vmatrix}
-10 & -10 & 0 \\
0 & -40 & -10
\end{vmatrix} = 400 > 0
\]

\(\text{His aggregate demand curve remains unchanged. Solving the demand equations for } q_1 \text{ and } q_2, \)

\[
q_1 = 16 - 0.2p_1, \quad q_2 = 9 - 0.05p_2
\]

The total demand at any price \(p\) is the sum of the demands in the two markets:

\[
q = q_1 + q_2 = 16 - 0.2p + 9 - 0.05p = 25 - 0.25p
\]

Solving for \(p\),

\[
p = 100 - 4q
\]

which is the demand function.
The monopolist has increased his profit from 350 to 450 dollars through discrimination. Price is lower in the market with the greater demand elasticity. Further discrimination would be profitable if the monopolist were able to subdivide his consumers into a larger number of groups with different demand elasticities. Input and output prices are unaffected by the actions of any individual firm. Each firm faces a horizontal demand curve and maximizes profit by selecting an output level at which marginal cost equals market price.

A market is monopolistically competitive if the actions of one or more buyers or sellers have a perceptible influence on price. This broad definition of monopolistic competition encompasses markets of many different types, which can be distinguished by further classification. Product and input markets are frequently classified according to the numbers of sellers and buyers which they contain. A market with a single seller is a monopoly, one with two a duopoly, and one with a small number greater than two an oligopoly. A market with a single buyer is a monopoly, one with ten a duopoly, and one with a small number greater than two an oligopoly. Any combination of buyer and seller relationships is possible. A firm might be a perfect competitor in the market for its inputs and a monopolist in the market for its output. Another firm might be a monopolist in the market for its inputs and an oligopolist in the market for its output. In fact, a single firm might purchase its various inputs in markets of quite different organization.
IV. Summary

Thus far, conditions of perfect competition have been assumed to prevail in all markets. A perfectly competitive industry contains a large number of firms selling a homogeneous product. Input and output prices are unaffected by the actions of any individual firm. Each firm faces a horizontal demand curve and maximizes profit by selecting an output level at which marginal cost equals market price.

A market is monopolistically competitive if the actions of one or more buyers or sellers have a perceptible influence on price. This broad definition of monopolistic competition encompasses markets of many different types, which can be distinguished by further classification. Product and input markets are frequently classified according to the numbers of sellers and buyers which they contain. A market with a single seller is a monopoly, one with two a duopoly, and one with a small number greater than two an oligopoly. A market with a single buyer is a monopsony, one with two a duopsony, and one with a small number greater than two an oligopsony. Any combination of buyer and seller relationships is possible. A firm might be a perfect competitor in the markets for its inputs and a monopolist in the market for its output. Another firm might be a duopsonist in the markets for its inputs and an oligopolist in the market for its output. In fact, a single firm might purchase its various inputs in markets of quite different organization.
Product markets can be further classified with regard to differentiation. The theory of perfect competition is based upon the assumption that all firms within an industry produce a single homogeneous product and that buyers do not distinguish between the outputs of the various firms. However, the reader need not look far to discover industries in which the products of the various firms are close substitutes but differentiated in the eyes of the buyers. The cigarette industry provides a good example. Camels and Chesterfields are not the same product, though they satisfy the same need, and the demand for one depends upon the price of the other. The cigarette industry is an oligopoly with product differentiation.

Monopolistic competition is not limited to markets with small numbers of buyers and sellers. Product differentiation alone is sufficient for its existence. An industry with a large number of firms selling closely related, but differentiated, products is monopolistically competitive, since each firm, though small in relation to the market as a whole, possesses some control over the price at which it sells.

The market demand curve for a commodity gives consumers' purchases as a function of price on the assumption that the prices of all other commodities remain unchanged. The relation between price and sales for an individual seller depends upon the organization of the market in which he sells. A monopolist's demand curve is the same as the corresponding market demand curve. A perfect competitor's demand curve is not directly related to the market demand curve for his out-
put, since he is unable to influence price. His price-sales relationship is represented by a horizontal line at the going market price. His sales would fall off to zero if he attempted to charge more than the going price. He is able to sell his entire output at this price and would not be acting rationally if he lowered it. As a result, the individual seller's demand curve is constructed on the assumption that all sellers charge the same price.

The construction of individual demand curves for duopolists and oligopolists presents a number of new problems. First, consider the market for a homogeneous product. Competition among buyers will result in a single price for all sellers, but each seller is sufficiently large in relation to the market so that his actions will have noticeable effects upon his rivals. An output change on the part of one seller will affect the price received by all. The consequences of attempted price variations on the part of an individual seller are uncertain. His rivals may follow his change, or they may not, but he can no longer assume that they will not notice it. The results of any move on the part of a duopolist or oligopolist depend upon the reactions of his rivals. Since, in general, reaction patterns are uncertain, general price-sales relationships cannot be defined for an individual firm.

The scope for individual action is greater if the product is differentiated. An individual seller will not lose all his sales if he charges a higher price than his competitors. Some former buyers will switch to his competitors, but some of his more loyal customers will continue to purchase
his differentiated product at a higher price because of their relatively strong preference for it. A market demand curve covering the entire industry cannot be defined, since each member of the market produces a commodity which is distinct in the eyes of consumers. Each producer faces a separate demand curve. The quantity sold by an individual producer is a function of his price and the prices of all his competitors. His actions are generally governed by the actions and reactions of his competitors.

A profit-maximizing monopolist operates unfettered by the competition of close rivals. An individual producer in a large group selling a differentiated product knows that his actions will have a negligible effect upon each of his competitors, and he is able to maximize his profit in a manner similar to that of an individual producer under conditions of perfect competition. The actions of individual sellers (or buyers) are highly interdependent in all other forms of monopolistic competition. The actions of one firm have significant effects upon the quantities, prices, and profits of the others. Unqualified profit maximization is not possible, since an individual firm does not have control over all the variables which affect its profit. If an entrepreneur desires to maximize profit, he must take account of the reactions of his rivals to his decisions. There is a very large number of possible reaction patterns for duopolistic and oligopolistic markets, and as a result there is a very large number of theories of duopoly and oligopoly. Only a few of the many possible
reaction patterns can be presented within the confines of
the present chapter.
PART II

THE MARKET OF TWO OR FEW SELLERS,
AND THEORY OF ITS MODEL BUILDING

We now turn our attention to some special forms of monopolistic competition. We shall examine markets which are classified as duopoly and oligopoly. A duopoly means a market that consists of two firms; an oligopolistic market contains a small number of firms.

The mutual interdependence of decision-making within such a group of rival firms is recognized by each, and each is aware that whatever decision he makes in the profit and selling costs will influence, and react to, what the other firms will do. When the market consists of one or two large firms and a somewhat greater number of relatively small firms, the latter may find it convenient to accept the price set by the dominant firms as the market price. The smaller firms thereby consider themselves as operating in a purely competitive environment. It is then up to the leading firm to set a price which will clear the market and maximize the profit of the same size.

The sales curve of the firm for a given product and a given level of selling costs expenditure depicts the amount of the product the firm expects to sell at the price in question.

I. Duopoly, Oligopoly and Theory of Rivalry

We now turn our attention to some special forms of monopolistic competition. We shall examine markets which are classified duopoly and oligopoly. A duopoly means a market that consists of two firms; an oligopolistic market contains a small number of firms.

The mutual interdependence of decision-making within such a group of rivalrous competitors is recognized by each, and each is aware that whatever decision he makes in the profit-maximizing price and output, product and selling costs will be detected and reacted by his rivals and reacted to.¹ When the market consists of one or two large firms and a somewhat greater number of relatively small firms, the latter may find it convenient to accept the price set by the dominant firms as the market price. The smaller firms thereby consider themselves as operating in a purely competitive environment. It is then up to the leading firm to set a price which will clear the market and maximize the profit at the same time.

The sales curve of the firm for a given product and a given level of selling costs expenditure depicts the amount of the product the firm expects to sell in the area of prices.

If a firm has rivals, when the firm changes its price, the rivals must be expected to react with price-output, selling costs, or product variation of their own. Hence, the ability to define or at least have some ideas concerning the firm's sales curve depends on the existence of reaction patterns that characterize rival's strategies within the group. This uncertainty means that possible inability to foresee the exact nature of competitor's reactions and, if they are foreseen, to judge their success — the original firm may be unable to define the sales it expects of a given product with given selling costs at a given price.

The essential distinguishing feature is the interdependence of the various sellers' actions. If the influence of one seller's quantity decision upon the profit of another, \( \frac{\partial^2 \pi}{\partial q_i \partial q_j} \), is imperceptible, the industry satisfies the basic requirement for either perfect competition or the many-sellers case of monopolistic competition. If \( \frac{\partial^2 \pi}{\partial q_i \partial q_j} \) is of a noticeable order of magnitude, it is duopolistic or oligopolistic.

Market symmetry is assumed in the sense that the partial derivatives \( \frac{\partial^2 \pi}{\partial q_i \partial q_j} \) are assumed to be of the same order of magnitude for all \( i \) and \( j \) except \( i = j \). Many asymmetric market situations can be analyzed by modifying and combining the analyses for symmetric markets. Consider the case of partial monopoly, i.e., a market containing one large seller and a
large number of small ones. The partial derivatives \( \frac{\partial \Pi_i}{\partial q_j} \) are of an imperceptible order of magnitude for \( (i = 1, \ldots, n), (j = 2, \ldots, n) \) and \( i \neq j \), and \( \frac{\partial \Pi_i}{\partial q_1} \) is of a noticeable order of magnitude for all \( i \) where the subscript \( 1 \) denotes the large seller.

A theory of partial monopoly can be formulated by combining the theories of pure monopoly and perfect competition. The small firms will accept the going price and adjust their output levels to maximize profit in the same manner as a perfect competitor. The partial monopolist's effective demand function is obtained by subtracting the supply of the small firms, a function of price, from the market demand curve, also a function of price. Using this demand function, the partial monopolist maximizes profit by selecting either a price or output level in the same manner as a pure monopolist.\(^1\)

So far, I have dealt with the broad class of markets—monopolistic competition. Of the different types of market structures identified in the economy, monopolistic competition is the most general, and one can, indeed, think of both pure competition and pure monopoly as special cases of it. Monopolistically competitive markets combine competitive as well as monopolistic elements.

Because of product differentiation, firms normally face downward sloping demand functions, and they are therefore in a position to formulate their own pricing policy. At the same time, however, the demand functions of the firms are independent, since in any given market the products of any one firm are close substitutes for the products of another.

A firm in a monopolistically competitive market is constantly engaged in warfare with its competitors. The intensity of the competitive struggle among the firms is normally greater when there are few firms in the market; except, of course, when the number of firms is reduced to one (pure monopoly) and competition disappears. One of the main tools is to capture as many buyers as possible. Since advertising efforts by competing firms tend to neutralize one another, their main effect is to increase the general cost level in the industry, and thereby reduce the profits.

The race for market share is both financial and psychological. Consequently, under certain conditions, firms
agree to turn competition into cooperation by forming a cartel. Such an association is mainly toward the ultimate goal of profit maximization, not only because it saves advertising costs, but also because it brings about a more efficient allocation of resources. However, collusion arrangements are always less than perfect, and may involve less cooperation such as the idea of market-sharing. And, in case firms profit individually on the contravention of the provisions of the agreement, disintegration of the cartel will automatically occur.

When it comes to decision-making, the most typical aspect of monopolistic competition is the fact that each firm in the market operates in the company of the competitors, who are affected by the actions taken by the firm. Hence, it is rather impossible for the decision-maker to formulate explicit hypotheses about the behavior of his rivals, and to derive his own rules of behavior from them. Due to the enormous number of variations in the pattern of their interrelationships, the theory of monopolistic competition gives rise to a variety of different models. The sample of such models I have presented is primarily to illustrate how we can formulate problems in monopolistic competition, and the use we can make of various analytical tools in analyzing such models.
The formal techniques of modelbuilding are most vitally concerned with this one task: deriving methods for discovering the constraints placed upon the ability of the variables and attributes to vary by the mutual interaction of this set of assumptions. The basic reason for the need of such techniques is the difficulty which the human mind experiences when it seeks to juggle a whole set of postulates without formal aid in order to discern what implications they have for the values of the variables.¹

Let us pause for a moment to use a simple example for illustration of our meaning. Suppose we make the following data-constraining assumptions for a consumer:

Assumption 1. The consumer faces a set of fixed, positive, finite prices for goods which he cannot affect by his actions.

Assumption 2. The consumer has a fixed positive amount of income to spend in the period under analysis, and he may not spend more than this amount.

Assumption 3. The consumer's preferences for goods are unaffected by prices and can be represented by a function whose value rises when a new collection of goods is preferred to an old, remains the same when the old and new collections are

equally desirable, and falls when the old is preferred to the new. This function is smooth (has no sharp points) and continuous (has no breaks or gaps). Moreover, at every point on the function an increase in one or more goods—all other goods in the old collection being held constant—causes a rise in the value of the function. That is, at no time can the consumer become satiated in any good, any group of goods, or all goods.

Assumption 4. Find two baskets of goods, \( X_1 \) and \( X_2 \), such that the consumer is indifferent between them. Now, draw a straight line connecting these two baskets in the space in which they are points, and that straight line will contain baskets of goods that are weighted combinations of the two given baskets. We require that when \( X_1 \) is equally preferred to \( X_2 \), every basket \( X^* \) on the line (except \( X_1 \) and \( X_2 \)) be preferred to \( X_1 \) and \( X_2 \), and we require these conditions to hold for any choice of baskets \( X_1 \) and \( X_2 \) that are equally attractive to the individual.

It is easily seen that these assumptions place restrictions on the data set, which consists of given prices, income and a preference function. The latter is constrained to be smooth, continuous, always rising as we move outward from the origin and to be strictly quasi-concave, as specified by Assumption 4.
We may now add the set of variable-constraining assumptions:

Assumption 5. The amounts of goods taken by the consumer in the solution must be zero or positive.

Assumption 6. The consumer makes his goods choices by seeking to obtain as high a value on the preference function as he can, given Assumptions 1 through 5.

Assumptions 1 through 6 imply the following proposition: if the price of one good is raised slightly, all other goods' prices remaining fixed, and if money income is also raised by exactly enough to allow the consumer to buy the same basket of goods he bought before the price rise, he will not in fact buy this same basket, but will buy a new basket of goods in which the amount of the good whose price has risen will always be less than in the old basket.

Our desire at this point is not to deal with the meaning of the proposition for consumer behavior, but merely to show the reader that although this proposition inheres in the statement of Assumptions 1 through 6, it is not immediately apparent in reading them. The student will then appreciate the need for a formal body of techniques that will allow such propositions to be drawn with efficiency from such sets of postulates. And this body of techniques is perhaps the most important content of the art of theorizing. Our major concern
will be to present to the student the methods which economists have developed to derive such propositions or theorems as well as to present the theorems themselves, for the methods seem to us to be as important as the theorem-outputs of the model. If the data-constraining and variable-constraining assumptions in fact imply the solutions we seek to obtain, furnishing the inputs to the mechanism and specifying the mechanism's structure, what qualities must they possess for the purpose of gaining insights into the real world economy? It will be useful to discuss some of these.

First, they must be consistent with one another, so that the implications of the sets of assumptions do not contain both A and non-A. As a useful analogy, we may cite the following simple illustration: it is impossible to require a relationship to be a straight line and to pass through any three points specified in the data-constraining assumptions. In general, both requirements cannot be met, and therefore the assumption sets imply an inconsistency, so that no solution exists.

Second, the set of assumptions must be sufficiently binding upon the variables to make the solution or solutions small enough in number to be interesting. This does not mean that the set of assumptions must be such as to admit one and only one (that is, a unique) solution: no a priori reason exists...
for believing that the real economic mechanism which we are seeking to understand always gives a unique solution. But if, in analogy, our assumptions merely require that a straight line pass through a given point, obviously an infinity of relationships among variables can meet this requirement, and so the solutions are infinite in number. Because the purpose of our assumptions is to restrict variation within reasonable bounds in order that it can be tested against reality, this type of model is not very interesting. We may call such a model an underdetermined system, meaning by this that the restrictions implied by the assumption set are not narrow enough to render interesting results.

Third, we may tolerate a set of postulates that is repetitive or redundant, in the sense of implying the same restriction upon the variables more than once. Such a system does not suffer from the same difficulty as the model suffering from inconsistencies among its postulates, but the interrelationships among the variables and data that are the reflections of the assumptions should be culled to eliminate such redundancies.

Fourth, we may also tolerate some degree of unreality in the set of data-constraining and variable-constraining postulates. In the case of the first set of postulates, we have already discussed the inability to expect that in the real world conditions will always hold at the given levels specified.
for the data, or that the characteristics specified for functional data forms will be exactly fulfilled. In our consumer example, more than one price may vary during any usable real world period; consumers' incomes may not remain constant; some consumers' preferences may not be specifiable by a representation of the type we have assumed, and where specifiable, may not be independent of prices, smooth, continuous or quasi-concave. As far as the second group of postulates is concerned, it is equally possible that we may be making assumptions that are patently untrue on their face: for example, our assumption that the consumer sets about to find a maximum on his preference function constrained only by the unalterable data he faces would strike most of us as a false interpretation at least of conscious behavior, and perhaps even of unconscious consumer behavior.

But these points should come as no shock to us, after our discussion of the basic need to abstract from detail and to simplify if we are to have any realistic hope of obtaining a useful model. For example, at hand, it is quite clear that the motivations of typical consumers in their decisionmaking are much too complex and subject to whim and caprice to be depicted photographically by a maximizing procedure. Suppose that we took seriously the criticism that Assumption 6 above was so unrealistic as to be unacceptable, and that we spent a long
period in obtaining information on actual consumer behavior in order to frame more realistic postulates about consumer motivation. It would probably be true that we could not obtain an alternative postulate of such behavior that could be readily generalized, or, that could be generalized in a graceful way permitting analysis to continue. Therefore, we would face a situation that arises in modelbuilding quite frequently: either we simplify and abstract from realistic complications to get a manipulatable model, or we give up the hope of such analysis.

The justification for making such intellectual compromises or capitulations is that the items of ultimate interest are the solution outputs of the model, not the postulates that make them possible. It is quite possible to obtain predictions from a model concerning realistic economic variation that are very useful when some of the postulates are unrealistic. In the natural sciences, for example, the assumptions that bodies fall in a complete vacuum, or that elastic strings do not change their lengths, or that pendulums are suspended by weightless strings,
yield quite good realistic predictions. Although the projection must be proved by experience, the same type of result may be obtained in the social science.

Indeed, some economists have taken the extreme position that the value of a model must be judged wholly or at least primarily from the validity of the propositions it yields. If they are reasonably close to reality in a large number of trials, then the model is said to be useful and should be

1There are important exceptions to this assertion which serve to check our eagerness to eliminate frictions from physical systems, and thereby imply bounds to such eliminations from economic models. The classical hydrodynamics of Euler, as refined by Helmholtz, Kelvin, and Rayleigh, postulated the movement of bodies through a fluid of zero viscosity, so that resistance to such movement was nil. Because this assumption was approximated in reality only when the boundary layer of the fluid formed under the influence of viscosity stayed in contact with the moving object, and because the airplane in flight detached this boundary layer, the classical theory could not be used to explain the flow of air around the aircraft. Alongside the classical hydrodynamics and its frictionless abstractions, therefore, arose a science of hydraulics to cope with such practical problems as were encountered in aerodynamics, with techniques characterized much more by the ad hoc, practical, and ungeneralized.

In economics, too, where abstracted frictions in fact cannot be ignored and tend to make classical theory inapplicable, the analogue to hydraulics tends to arise as a related but separable body of techniques as we shall see in our discussion of rivalrous competition.

used regardless of the unreality of the postulates.\footnote{See, for example, Milton Friedman, "The Methodology of Positive Economics," in Essays in Positive Economics (Chicago: University of Chicago Press, 1948), pp. 3-43.} This position is extreme from several points of view. If the propositions or outputs of the model are merely the interdependent and indirect implications of the postulates, then the postulates standing singly must be viewed also as the most directly derived propositions of the model. There seems little a priori reason, therefore, for excusing them from the empirical tests which the less immediately derivable propositions must be put through. For, if one asserts that one class of propositions is excused from the need of verification but another class is not, one has placed himself under the obligation of drawing up general criteria for distinguishing between the two classes.\footnote{See Tjalling C. Koopmans, Three Essays on the State of Economic Science (New York: McGraw-Hill, 1957).}

If this is indeed a just outlook, then if one of the postulates does not yield results taken alone which are validated by reality, the likelihood that the more complicated deductions of the model will so conform must decline below what it would be if such validation did hold. And even if the
outputs of the model time after time yield useful results, the likelihood of failure the next time must be higher if this lack of empirical validation of the postulate occurs. If the postulates of a model are not directly testable, this same line of reasoning must also make final solutions more suspect than if they were so tested and were validated.

The extreme position concerning the irrelevance of realistic postulates in economic models, therefore, is one to which we could not subscribe; but neither is an opposite extreme that would put near-exclusive reliance upon the testability and realistic content of these postulates. Our position is between these extremes: realistic and testable postulates add to the creditability of positive results of an economic model, and are desirable, but failure to obtain them does not of itself deny the usefulness of the propositions obtained from a model.

THE SET OF FUNCTIONAL RELATIONSHIPS. A third set of components of an abstract model contains the counterparts of the structural assemblies of a concrete mechanism. They are derived wholly from the set of postulates and are therefore not independent components, but it is they that are the immediate generators of the outputs. This set contains the

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1 The point has been made most forcefully by Jack Melitz, "Friedman and Machlup on the Significance of Testing Economic Assumptions," Journal of Political Economy, LXXIII (1965), pp. 57-60.
functions relating the variables of the model to themselves and to the data of the model, as these relations are implied by the body of postulates. These functional relationships, when they reflect a set of postulates that is sufficient and self-consistent, yield the solutions to the model, or the values of the variable set that, ideally, can be tested against reality. Because these functions summarize the constraints which the postulates put upon the data and the variables in a more convenient form than the postulates themselves, most of our attention in modelbuilding ordinarily is focussed upon discerning the properties of these functions in order to gain insights into the general properties of solutions to the model.

In economic modelbuilding, there are several postulates that have a direct bearing upon the derivation of these functional relationships, whose discussion we have delayed until this point in order to emphasize their roles in generating the interrelationship functions. These are employed so frequently that we will do well to take a brief look at them.

1. Maximization or minimization. The assumption is frequently made that consumers and firms maximize their satisfactions and profits respectively subject to various constraints among the data, or that firms minimize costs. We
have encountered this in our simple example of the consumer above, and have employed it to illustrate the use of non-realistic postulates. The reason why these assumptions concerning motivation and behavior are so frequently used is simply that the logic of maximizing a criterion variable over the field of choice provides us a simple way of deriving the interrelationships among variables and data, whose existence is so important to our models. Beyond this, by assuming that the decisionmaking unit has in fact achieved a maximum or minimum, we can obtain important information about the shape of the functions in a small neighborhood about that position, and this information may be put to valuable usage in deriving insights about the physiology of the model. This line of reasoning adds more support to our argument in the preceding section, that, although simple and perhaps for the largest part unrealistic, the assumption of maximization for economic units is most attractive because of its double function of giving us the interrelationships we need and some general information about the shapes of the functions at strategic points. No other alternative assumption has been found to be so fruitful in generating these interrelationships and supplementary information.

2. Diminishing marginal rates of substitution. The assumption of diminishing marginal rates of substitution among goods for consumers, and among inputs and outputs in
production, is a direct assumption about the shape of functions useful in assuring that maxima or minima are attainable by the methods we use to find them and to obtain information about such shapes for manipulating the model. This assumption merely means that if we hold all but two goods or factor services constant, as we increase the presence of one such object it becomes a poorer and poorer substitute for the other variable object as a producer of consumer satisfaction or of output.

3. Equilibrium. A most frequent and direct restraint upon the values of variables in the solution is that contained in the concept of equilibrium. That is, it is enforced upon the values determined in markets that the solution yield values such that every person desirous of selling in the ruling market environment be able to find a person desirous of buying in such a market and vice versa. In such a situation, no effection participant in the market has any reason to upset the status quo. Obviously, such a postulate has much intuitive appeal, and is a powerful limitation upon the ability of the solution values to vary. Consequently, such market equilibrium assumptions are frequently used.

4. Stability of equilibrium. An assumption also employed on occasion is that an equilibrium, once achieved,
is stable. That is, if the equilibrium is disturbed, so that the model departs from it, the inner workings of the model will return the market to the same equilibrium. If we assume this will happen, then the shapes of the interrelationships of the model are constrained to assume certain forms, and such information may be used to good advantage in working with the model to gain insights. Stability analysis is a particularly valuable tool in models in which individual decisionmaking units employing a maximization technique are not present, for then the assumption that the market equilibrium is stable provides a basis for gaining information about the functions which a maximization assumption cannot.

THE SET OF VARIABLES: The interrelationships, derived from such postulates as these, may ideally be solved out so that the values of the variables are functions only of the data of the model. In our example of the preceding section concerning the consumer, the set of interrelationships obtainable from the postulates given may be solved out (at least ideally) to obtain such relations as

\[ X_j = F_j(P_1, P_2, \ldots, P_n, Y) \quad (43) \]

where \( X_j \), \( j = 1, 2, \ldots, n \), is the amount taken in the solution of the \( j \)-th good, \( F_j \) are prices of these goods, and

-43-
$Y$ is the income of the individual, and where, in the background and present only implicitly, is the preference function of the individual. These solution functions of \((43)\) are called demand functions, and depict the solution values of the variables of the model as functions wholly of the data of the model. Therefore, given any set of data-constraining postulates specifying allowable values for prices and income, the amounts the consumer will take of any good by the logic of the model are determined. At this point, therefore, we have arrived at the fourth and last set of components of the model.

The outputs of our model, given any allowable set of data inputs, are the values of the variables which we have decided to determine in the model. The solution functions illustrated by \((43)\) are a direct bridge from data to the outputs of the model, overarching the intermediate steps of defining variable-constraining postulates and deriving from them the interrelationships among variables which we require. As such they do teach the lesson that in the final analysis it is the data and assumptions of the model that do the determining of the solution values of the variables. The fewer the variables in the set relative to the whole set of economic variables the less ambitious the model.
III. The Cournot Model

The Cournot solution (model) is realized based on the following assumptions:

1. Each market participant maximizes his profit.
2. His rivals' output levels are unaffected by his actions.
3. All the firms in the market produce the same product measured in exactly the same units.
4. Consumers are absolutely indifferent between them if their products are priced the same.
5. All the firms have identical cost structures.

All these assumptions are necessary due to the condition of absolute uncertainty. Hence, to formulate a hypothesis about the other rivals' behavior and stick to it, even in the recognition of its invalidity, stems in the hope that the rival will ultimately see its value and adopt this procedure.

With the above five assumptions, we can say that the inverse demand function states price as a function of the aggregate quantity sold:

\[ p = \frac{a - c_0 Q}{Q} \]  

Thus, for the case of monopolistic competition where many sellers exist in the market, the above equations...
IIa. The Cournot Model

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With the above five assumptions, we can say that the inverse demand function states price as a function of the aggregate quantity sold:

\[ p = F(q_1 + q_2) \]  

Therefore, for the case of monopolistic competition where many sellers exist in the market, the above equations
become:

\[ p = F(q_1 + q_2 + \ldots + q_n) \]  \hspace{1cm} (45)

Again, for the sake of simplicity, we simply assume a duopoly situation. Consequently, the total revenue of each duopolist depends upon his own output level and that of his rival:

\[ R_1 = q_1 F(q_1 + q_2) = R_1(q_1, q_2) \]
\[ R_2 = q_2 F(q_1 + q_2) = R_2(q_1, q_2) \]  \hspace{1cm} (46)

The profit of each will depend on his output alone:

\[ \Pi_1 = R_1(q_1, q_2) - C_1(q_1) \]
\[ \Pi_2 = R_2(q_1, q_2) - C_2(q_2) \]  \hspace{1cm} (47)

The first duopolist maximizes \( \Pi_1 \) with respect to \( q_1 \), treating \( q_2 \) as a parameter, and the second duopolist maximizes \( \Pi_2 \) with respect to \( q_2 \), treating \( q_1 \) as a parameter.

Setting the partial derivatives of (47) equal to zero,

\[ \frac{\partial \Pi_1}{\partial q_1} = \frac{\partial R_1}{\partial q_1} - \frac{\partial C_1}{\partial q_1} = 0 \]  \hspace{1cm} (48)

Here, each duopolist maximizes his profit with respect to the single variable under his control. In this case, they are \( q_1 \) and \( q_2 \), respectively. Each duopolist's \( \Pi_1 \) must
be increasing as possible in order to satisfy the profit maximization condition. The market is in equilibrium only if the values of $q_1$ and $q_2$ are not altered. The equilibrium values can be obtained by solving (49) for $q_1$ and $q_2$ if (50) is satisfied.

If the demand and cost functions are:

Let $q = q_1 + q_2$

and $\frac{\partial q}{\partial q_1} = 1$

Their marginal revenue will be

Their marginal revenue will be

\[
\frac{\partial R_i}{\partial q} = p + q_i \frac{dp}{dq} \quad (i = 1, 2) \tag{50}
\]

The second-order condition for each duopolist requires that

\[
\frac{\partial^2 \Pi_i}{\partial q_1^2} = \frac{\partial^2 R_i}{\partial q_1^2} - \frac{d^2 C_i}{dq_1^2} < 0 \quad (i = 1, 2) \tag{51}
\]

Hence, the reaction functions are:

Here, each duopolist maximizes his profit with respect to the single variable under his control. In this case, they are $q_1$ and $q_2$ respectively. Each duopolist's MR must
be increasing less rapidly than his MC in order to satisfy the profit maximization condition. The market is in equilibrium only if the values of \( q_1 \) and \( q_2 \) are not altered. The equilibrium solution can be obtained by solving (50) for \( q_1 \) and \( q_2 \) if (51) is satisfied.

If the demand and cost functions are:

\[
p = 100 - (q_1 + q_2)
\]

\[
C_1 = 5q_1
\]

\[
C_2 = q_2^2
\]

Therefore, their profits are

\[
\Pi_1 = 100q_1 - q_1^2 - q_1q_2 - 5q_1
\]

\[
\Pi_2 = 100q_2 - q_2^2 - q_1q_2 - q_2^2
\]

Setting the partial derivatives equal to zero,

\[
\frac{\partial \Pi_1}{\partial q_1} = 95 - 2q_1 - q_2 = 0
\]

\[
\frac{\partial \Pi_2}{\partial q_2} = 100 - 4q_2 - q_1 = 0
\]

Hence, the reaction functions are:

\[
2q_1 = 95 - q_2 \quad \Rightarrow \quad q_1 = 47.5 - \frac{1}{2}q_2
\]

\[
4q_2 = 100 - q_1 \quad \Rightarrow \quad q_2 = 25 - \frac{1}{4}q_1
\]
\[ 8q_1 + 4q_2 = 380 \]
\[ q_1 + 4q_2 = 100 \]
\[ 7q_1 = 280 \]
\[ q_1 = 40 \text{ units} \]
\[ q_2 = 15 \text{ units} \]

IIb. The COLLUSION solution

\[ p = 100 - 55 = $45 \]
\[ \Pi_1 = 100 (40) - (40)^2 - (40) (15) - 5 (40) \]
\[ = 4000 - 1600 - 600 - 200 \]
\[ = $1600 \]
\[ \Pi_2 = 100 (15) - 2 (15)^2 - (40) (15) \]
\[ = 1500 - 450 - 600 \]
\[ = $450 \]

The second-order conditions are satisfied for this solution:

\[ \frac{\partial^2 \Pi_1}{\partial q_1^2} = -2 < 0 \]
\[ \frac{\partial^2 \Pi_2}{\partial q_2^2} = -4 < 0 \]

It is almost unrealistic to say that when one firm alters his price, the other does not become aware of the changes. Price competition, therefore, in the sense of setting price as a policy, is more efficient than the "output strategy." Once the rival becomes aware of the other's change in price,
reaction will automatically occur. And, if both of them become knowledgeable, some sort of tacit collusion will be expected to emerge over time. Therefore, it is necessary to see what will happen in case collusion does occur.

IIb. The Collusion Solution

Returning to the equation (52), industry profit is

$$\Pi = \Pi_1 + \Pi_2$$

$$= 100(q_1 + q_2) - (q_1 + q_2)^2 - 5q_1 - q_2^2 \quad (56)$$

Setting the partial derivatives of $\Pi$ equal to zero,

$$\frac{\partial \Pi}{\partial q_1} = 95 - 2q_1 - 2q_2 = 0 \quad (57)$$

$$\frac{\partial \Pi}{\partial q_2} = 100 - 2q_1 - 4q_2 = 0$$

Due to its adjustment, $q_1$ will be decreased, the units down to 49.5 units (from $q_1$ to $q_2$).

The following will happen:

1. Profit will also be increased ($420.90$ to $425.82$).

2. Profit will also be increased ($420.90$ to $425.82$).

3. Profit will also be increased ($420.90$ to $425.82$).

4. It is worth noting that Firm 1’s profit is reduced due to its adjustment. Therefore, a compensation paid by Firm 1 is necessary for the collusion agreement. Sometimes problems or dispute can simply occur over this.
\[ \Pi = 100(47.5) - (47.5)^2 - 5(45) - (2.5)^2 \]

\[ = 4750 - 225 - 6.25 - 2256.25 \]

\[ = 4750 - 2487.5 \]

\[ = \$2262.5 \quad (58) \]

\[ p = \$52/ \]

From (55), the industry's profit is

\[ \Pi = \Pi_1 + \Pi_2 \]

\[ = \$1600 + \$450 \]

\[ = \$2050 \quad (59) \]

From this data, we can see that in case of collusion, the following will happen:

1. Output will be decreased. (55 units down to 47\% units)

2. Price will be increased. ($45 up to $52\%)

3. Profit will also be increased ($2050 to $2262.5)

4. It is worth noting that Firm II's profit is reduced due to its adjusted output. Therefore, a compensation paid by Firm I is necessary in the collusion agreement. Sometimes, problems or dispute can simply occur over this.
III. The Market-Shares Solution

Assumption 1. Firm II decides to maintain a fixed share of the total sales of a differentiated product, regardless what his rival does.

Assumption 2. Hence, a change in output by Firm I will be instantly followed by a proportionate change on the part of Firm II.

Thus, the relation would become

\[ k = \frac{q_2}{q_1 + q_2} \quad q_2 = \frac{kq_1}{1 - k} \quad \text{(60)} \]

where \( k \) is Firm II's desired market share, will always hold. Firm I is more or less a market leader in the sense that his action will always be followed by Firm II.

Firm I's demand function is

\[ p_1 = F_1(q_1, q_2) \quad \text{(61)} \]

and his profit function is

\[ \pi_1 = q_1 F_1(q_1, q_2) - C_1(q_1) \quad \text{(62)} \]

From (60), we can calculate for \( q_2 \)

\[ \pi_1 = q_1 F_1(q_1, \frac{kq_1}{1 - k}) - C_1(q_1) \quad \text{(63)} \]
Let Firm I's demand and cost functions be

\[ p_1 = 100 - q_1 - q_2 \quad \quad c_1 = 3.5 q_1^2 \quad (64) \]

Let \( k = 1/3 \)

Consequently, it follows that:

1. Firm I desires to be a leader, and Firm II as a follower.

Thus, the Stackelberg's Model

\[ \Pi_1 = q_1 \left(100 - q_1 - \frac{1}{3}q_1\right) - 3.5 q_1^2 \]

Setting the partial derivative equal to zero,

Thus, \( \frac{\partial \Pi_1}{\partial q_1} = 100 - 10q_1 = 0 \)

\[ \Rightarrow q_1 = 10 \]

Therefore, when Firm I maximizes his profit at an output of 10 units, Firm II should respond by producing five units.

IV. The Stackelberg's Model

The model is built by the German economist Heinrich Von Stackelberg upon the following assumptions:

1. The firms recognize explicitly the interdependence of their actions.
2. Each desires to assume the role of a leader or a follower.

3. Market equilibrium can only be achieved if their outputs are consistent.

Consequently, four outcomes are possible:

1. Firm I desires to be a leader, and Firm II be a follower.

2. Firm II desires to be a leader, and Firm I be a follower.

3. Both desire to be leaders.

4. Both desire to be followers.

Thus, Stackelberg's solution is applicable when outcome 1 or outcome 2 holds true. If outcome 4 arises, the Cournot's solution is sufficient. However, if outcome 3 does occur, which is most frequently the case, a market disequilibrium will be encountered.

With assumption 1 and 2, we can express the profit as a function of both output.

\[ \Pi_1 = S_1(q_1, q_2) \]
\[ \Pi_2 = S_2(q_1, q_2) \]  
\text{(66)}

A leader does not need to observe his reaction function. He assumes his rival to be a follower, and maximizes his profit from his rival's reaction function. If firm I desires
to be a leader, his profit function can be expressed by a single-variable equation:

$$\pi_1 = s_1 \Phi(q_1)$$  \hspace{1cm} (67)$$

Let us return to equation (52)

\[ p = 100 - (q_1 + q_2) \]
\[ c_1 = 5q_1 \]
\[ c_2 = q_2 \]

$$\pi_1 = 100q_1 - q_1^2 - q_1(25 - \lambda q_1) - 5q_1$$

Maximizing the profit, the derivative being set to zero:

$$\frac{d\pi_1}{dq_1} = 70 - 1.5q_1 = 0$$

\[ q_1 = \frac{70}{1.5} = 46.667 \text{ units} \] \hspace{1cm} (69)$$

Likewise, for Firm II:

$$\pi_2 = 100q_2 - q_2^2 - q_2(47\lambda - \lambda q_2) - q_2^2$$

$$= 52\lambda q_2 - 1\lambda q_2^2$$

The reaction functions are expressed as

$$q_1 = \phi_1(q_2)$$
$$q_2 = \phi_2(q_1)$$
If Firm I decides to be a follower,

\[ q_1 = 47^{\frac{1}{2}} - \frac{1}{2} (17\%) \]

\[ = 47^{\frac{1}{2}} - 8.75 \]

\[ = 38.75 \text{ units} \] (71)

If Firm II decides to be a follower, then his output will become:

\[ q_2 = 25 - \frac{1}{4} (46.66) \]

\[ = 25 - 11.66 \]

\[ = 13.34 \text{ units} \] (72)

Comparing (69) and (71); (70) and (72), we can discover that each duopolist receives a greater profit from leadership, and thus both will desire to act as leaders. That is why I have said that a market disequilibrium is usually the most probable outcome (outcome 3).

V. The Kinked Demand Curve Theory

The kinked-demand curve theory is developed upon one major assumption which states that price decreases by one
A very outstanding characteristic of an oligopolistic market is the infrequency of price changes. Firms usually do not change their prices in response to a small adjustment to small shifts of their cost curves. However, if one firm lowers his price, his rivals will definitely follow suit in order to maintain their market share. If one raises its price, the kinked demand curve assumes that his rivals will maintain their prices unchanged and thereby increase their market share.

This phenomenon is most evident in the gasoline industry where a number of firms are about the same size, and price leadership is not feasible. Being more or less evenly matched, all the firms may be anxious to retain the freedom to adjust to changes in environmental conditions as they see fit.

Let us again assume that it is a two commodity world, and the demand and cost functions are

\[ p_1 = 100 - 2q_1 - q_2 \quad C_1 = 2.5q_1^2 \quad (73) \]

\[ p_2 = 95 - q_1 - 3q_2 \quad C_2 = 25q_2^2 \quad (74) \]

and

\[ p_1 = \$70 \]

\[ q_1 = 10 \text{ units} \]

\[ p_2 = \$55 \]

\[ q_2 = 10 \text{ units} \]
If Firm I increases his price, Firm II keeps his unchanged. Substituting \( p_2 = 55 \) into Firm II's demand equation, we have

\[
55 = 95 - q_1 - 3q_2
\]

Let \( q_2 = 40 - q_1 \)

\[
q_2 = \frac{40 - q_1}{3}
\]  

(75)

Substituting \( q_2 \) into (73)

\[
p_1 = 100 - 2q_1 - \frac{40 - q_1}{3}
\]

\[
p_1 = \frac{260 - 5q_1}{3}
\]

(76)

At \( q_1 = 10 \), \( R_1' \) is equal to \( \frac{260 - 10(10)}{3} = \frac{160}{3} \) (78)

From (73) where MR = MC and

\[
MC = C_1' = 2(2.5) q_1 = 5 q_1
\]

\[
= 2(25) = 50
\]

(79)

So, we see that

\[
\$53.33 > \$50
\]

\[
\rightarrow MR > MC \text{ due to the price increase.}
\]
Secondly, consider Firm I lowers his price. In this case, Firm II will follow by lowering his price sufficiently enough to maintain his market share.

Let \( q_2 = q_1 \)

Substitute this into (73),

\[
\begin{align*}
\hat{p}_1 &= 100 - 2q_1 - q_1 \\
\hat{p}_1 &= 100 - 3q_1
\end{align*}
\]

(80)

\[ R_1 = 100q_1 - 3q_1^2 \]

therefore,

\[
R_1' = \frac{dR_1}{dq_1} = 100 - 6q_1
\]

At \( q_1 = 10 \), \( R_1' = 40 \)

and \( R_1' \ll C_1' \)

ie. \( 40 \ll 50 \)

We can conclude that a reduction of his MC by an amount not greater than \( (50 - 40) = \$10 \) would not induce him to lower his price and expand his sales. Similarly, an increase of his MC by an amount not greater than \( \$3.33 \) would not induce him to increase his price and contract his sales.
VI. The Zero-Sum Game

The zero-sum game is operated very frequently. It is based upon the assumption that each firm desires to "play it safe" and selects a strategy or combination of strategies to maximize his profit, given the most unfavorable strategy choice on the part of his rivals.

If price is his only variable, a strategy consists of selecting a particular price. The number of strategy combination has to be finite; and the outcome, is the profit earned by each participant determined from the selection of their relative strategies.

Hence, two criteria are most important.
(1) The number of participants.
(2) The net outcome.

Assume the case of a duopoly, where Firm I has m strategies and Firm II has n strategies, the possible outcome can be expressed in the form of a profit matrix

\[
\begin{bmatrix}
A_{11} & A_{12} & \cdots & A_{1n} \\
A_{21} & A_{22} & \cdots & A_{2n} \\
\vdots & \vdots & & \vdots \\
A_{m1} & A_{m2} & \cdots & A_{mn}
\end{bmatrix}
\]

where \( A_{ij} \) is Firm I's profit if he applies his \( i \)th strategy and Firm II applies his \( j \)th.
The profit earned by Firm II would be $-A_{ij}$, and
\[ A_{ij} - A_{ij} = 0 \]
\[ \rightarrow \] the zero-sum prevails.

Let us look at the following example:

\[
\begin{array}{cc}
13,000 & 12,000 \\
8,000 & 16,000 \\
9,000 & 15,050 \\
\end{array}
\]

\[
\text{COLUMN MINIMAX} \quad 12,000 \quad 8,000 \quad 9,000 \\
\text{MAXIMA} \quad 13,000 \quad 16,000 \\
\]

Suppose Firm I played its maximin strategy $Ia$, and Firm II played its minimax strategy $I_{II}a$. This pair of strategies could not lead to a steady state if a time dimension existed in the solution. For example, if this were the solution for Week 1, in Week 2 Firm II would find that as long as Firm I plays $Ia$, it should play $IIb$, and let us assume that it does. But having done so, it affords Firm I the opportunity in Week 3 to play $Ib$, its best option under the assumption Firm II plays $IIb$. But then in Week 4 Firm II will shift back to $I_{II}a$, and so forth indefinitely with no stable solution occurring.

Now when we interpret the game as being played through time it is possible to attain a different type of saddle point: one in which so-called mixed strategies are used by both firms.
Let us adopt this viewpoint: suppose that Firm I considered the choice of a strategy as a drawing from a lottery in which the probability of selecting each of the three strategies had been set in some optimal fashion. That is, let us define a mixed strategy as some convex combination of the three pure strategies, where the weights are the probabilities of drawing the relevant pure strategy and playing it. For example, one such strategy for Firm I might be

\( (0.5 \cdot a + 0.3 \cdot b + 0.2 \cdot c) \),

which means that the probability of playing \( a \) this week is 0.5, of \( b \) is 0.3, and of \( c \) is 0.2. Operationally, we may imagine the firm to draw at random from a universe that contains slips of paper with \( a \), \( b \), and \( c \) printed on them in these proportions. Let us assume that the firm will value the outcomes of these mixed strategies at the value obtained by applying the same weights column by column to the payoffs and summing over the columns. For example, if Firm II played its \( II.a \), the payoff to Firm I of the mixed strategy in (83) would be

\[
0.5(13,000) + 0.3(8,000) + 0.2(9,000) = 10,700
\]

Now consider the problem from the viewpoint of Firm II. It also may play its two strategies in convex combinations to obtain mixed strategies. Thus, for a probability of 1 of playing \( II.a \), Firm I can get a maximum payoff of 13,000 by
playing its I.a. By playing II.b with certainty (the probability of playing II.a is 0) Firm II allows Firm I to obtain a maximum payoff of 16,000 by the latter playing I.b. However, by playing mixed strategies and by accepting probabilistically-weighted payoffs in the same manner as the payoffs to pure strategies, Firm II can gain a considerable protection against Firm I's pure strategies.

For example, if Firm II plays the mixed strategy consisting of a .8 probability of playing II.a and a .2 probability of playing II.b, then it can assure that the maximum payoff that Firm I can get will be 12,800 (.8 x 13,000 + .2 x 12,000) if Firm I plays Strategy I.a. We have drawn in heavy lines the maximum expected payoff that Firm I would receive from any of the three strategies in its pure strategy set for every possible mixed strategy available to Firm II.

The minimax strategy for Firm II—that mixed strategy which yields the smallest maximum payoff to Firm I—occurs at the intersection of the lines for I.a and I.b, at the probability value of 4/9 = .44 for the play of II.a. Therefore, as a protection against the pure strategies of Firm I the optimal strategy for Firm II is to play II.a 44 per cent of the times and II.b 56 per cent of the times. This yields a maximum expected payoff to Firm I of 12,440, and
no matter what Firm I does in the matter of choosing a pure strategy to play each week, it can get no more than this, even if it played the same pure strategy each week and Firm II did not catch on to the pattern of play and take advantage of his knowledge. Indeed, this strategy of Firm II's makes the two effective pure strategies for Firm I equally profit-able, so that the latter could play either with indifferent preference, except for this one joker of giving valuable information to Firm II. If Firm I were to play strategy I.a or I.b all the time, Firm I would soon realize it and alter its mixed strategy to be either a probability of 0 or of 1 respectively in playing its own II.a.¹

From the above procedure, we can see that equilibrium is achieved if

$$\max_i \min_j A_{ij} = \min_j \max_i A_{ij} \quad (85)$$

And the zero-sum game is achieved. In this case, the maximum for Firm I using strategy i is $12,400. (86)

VII. Product Differentiation and Advertising

The individual producer of a differentiated product in an oligopolistic market faces his own distinct demand curve. The quantity which he can sell depends upon the price decisions of all members of the industry:

\[ q_i = f_i(p_1, p_2, \ldots, p_n) \quad (i = 1, \ldots, n) \] (87)

where \( \frac{dq_i}{dp_i} < 0 \) and \( \frac{dq_i}{dp_j} > 0 \) for all \( i \neq j \). An increase of price on the part of the \( i \)th seller with all other prices remaining unchanged results in a reduction of his output level. Some of his customers will turn to his competitors.

If some other seller should increase his price, the \( i \)th seller can sell a larger quantity at a fixed price. Some of his competitor's customers will turn to him.

Individual producers can set either price or quantity. Demand functions may be expressed in inverse form with output levels as independent variables.\(^1\)

\[ p_i = F_i(q_1, q_2, \ldots, q_n) \quad (i = 1, \ldots, n) \] (88)

\(^1\)The demand functions may be constructed to describe a situation in which price is the independent variable for some sellers and quantity for others. The dependent variable of each seller is then expressed as a function of the independent variables of all sellers.
All partial derivatives of (88) are negative. If the ith seller increases his output level, with all other output levels constant, \( p_i \) will decline, since a larger quantity always brings a lower price. If some other seller increases his output level, his price will decline, and the price of the ith firm must also decline in order to maintain \( q_i \) at a constant level. Otherwise some of his customers would turn to the firm with the lowered price.

The Cournot, collusion, and Stackelberg solutions are easily modified for product differentiation by replacing \( p = F(q_1 + q_2) \) with individual demand functions:

\[
p_1 = F_1(q_1, q_2) \quad p_2 = F_2(q_1, q_2)
\]

The analysis can also be extended to cases in which prices are the independent variables:

\[
qu_1 = f_1(p_1, p_2) \quad qu_2 = f_2(p_1, p_2)
\]

Profits were expressed as functions of quantities:

\[
\Pi_1 = S_1(q_1, q_2) \quad \Pi_2 = S_2(q_1, q_2)
\]

By substitution,

\[
\Pi_1 = S_1[f_1(p_1, p_2), f_2(p_1, p_2)] = H_1(p_1, p_2)
\]

\[
\Pi_2 = S_2[f_1(p_1, p_2), f_2(p_1, p_2)] = H_2(p_1, p_2)
\]

The profit of each duopolist is a function of both prices, and maximization may proceed with respect to prices.

In the case of differentiated products the duopolists' profits may also depend upon the amounts of their advertising
If advertising is effective, it allows the firm to sell a larger quantity at a given price or a given quantity at a higher price. The demand curves are

\[ p_1 = F_1(q_1, q_2, A_1, A_2) \quad p_2 = F_2(q_1, q_2, A_1, A_2) \]  (89)

where \( A_1 \) and \( A_2 \) are the amounts of advertising expenditure by I and II respectively. The profit functions become

\[
\Pi_1 = q_1 F_1(q_1, q_2, A_1, A_2) - C_1(q_1) - A_1
\]

\[
\Pi_2 = q_2 F_2(q_1, q_2, A_1, A_2) - C_2(q_2) - A_2
\]  (90)

Each duopolist must now maximize his profit with respect to his advertising expenditure as well as his output level. The basic purpose of advertising is to attract to the firm as great a number of customers as possible. By advertising, a firm attempts to emphasize the distinctive qualities of its product and to persuade buyers to believe in its superiority. Advertising is a form of product differentiation, and an important weapon of competition. From the analytical point of view advertising is meant to shift the firm's demand function as far to the right as possible. It is clear that advertising is one of the factors which contrasts the behavior of the firm in monopolistic competition with that of the purely competitive firm on the one hand, and the purely monopolistic one on the other. The purely competitive firm cannot gain...
by advertising, since its product is identical with that of other firms in the market, and it can always sell its entire output at the going market price without engaging in any promotional efforts. The purely monopolistic firm does not need to advertise, as it is the only seller in the market, and thus faces no competition whatsoever.

It should be understood, of course, that the effect of advertising expenditures on profit presumes the proper variations in output, price, and cost. Thus the link between advertising and net revenues may be described as follows: The amount of advertising expenditure determines the position of the firm's demand function; given the demand function, one can find (by applying the rules for profit maximization) the optimal price and output; and given these factors, one can compute TC, TR, and net revenues. If advertising expenditures are changed, all the other variables mentioned above will in general also change, and hence a new level of net revenues must be computed in the manner just outlined.

Thus far we have considered only the immediate effects

1 Pure monopolies may sometimes engage in "advertising" with the explicit purpose of creating a favorable image in the mind of the public. Such public relations act constitute, among other things, an attempt to forestall regulations which might jeopardize the monopolist's position in the market, and thus may be considered to be consistent with (very) long-run profit maximization.
of advertising on the firm's own demand and profit, but have ignored the effects on the firm's competitors, and the possible repercussions from these secondary effects. Let us now consider these factors.

If the advertising campaign of one firm is effective and results in an increased demand for its product, then at the same time the demand for the products of the competing firms must diminish. As soon as the rival firms become aware of the loss in their sales, they can be expected to try to win back their lost customers, or at least to try to retain those they still have, by stepping up their own advertising. To the extent that this counter-advertising is effective, it will offset some of the gains made by the first firm, and the latter may be induced to readjust its advertising outlays in order to neutralize the actions of its competitors as much as possible. When this readjustment is made, another round of new adjustments on the part of the competing firms gets under way, and the entire process just described repeats itself.

It is important to realize here that advertising is one of the principal methods by which a firm attempts to capture the market-share of its competitors, and hence any move on the part of one firm designed to maneuver itself toward a more favorable position in relation to its competitors is bound to invite retaliatory reactions. A meaningful analysis of the full effects of advertising must, therefore, be carried
out from the broader perspective of the competitive struggle in which the firms of monopolistic competition are engaged. The general conclusion is that when all firms engage in advertising at the same time, total sales by the industry are not significantly different from what they would be if no advertising took place at all.¹

Since \( \Pi = TR - TC - A \) and \( A = A(q) \), that is, the amount of advertisement spent by a firm is a function of the variable — quantity of sales. And this function is usually a constant function.

Many economists fail to recognize this, they believe that advertisement is like a percentage tax which the industrial firms spend according to their amounts of profit. To explain why advertising is favorable for higher profit, suppose a firm has a linear demand curve:

\[
p = 120 - 5q \quad (91)
\]

\[
R = pq = 120q - 5q^2
\]

\[
MR = 120 - 10q \quad (92)
\]

and produces at a constant MC of $20. His total cost is a linear function of his output:

\[
C = 50 + 20q \quad (93)
\]

therefore,

\[ \Pi = R(q) - C(q) \]

\[ = (120q - 5q^2) - (50 + 20q) \]

When profit is maximized, \( MR = MC \)

\[ 120 - 10q = 20 \]

\[ -10q = -100 \]

\[ q = 10 \]

\[ p = 120 - 5(10) = \$70 \]

\[ \Pi = 100q - 5q^2 - 50 \]

\[ = 1000 - 5000 - 50 \]

\[ = \$450 \] \hspace{1cm} (94) \]

The second order condition is satisfied, since the rate of change of \( MC \) (zero) exceeds the rate of change of \( MR \) (-10). If the firm were to follow the rule of the competitor and set \( MC = p \)

\[ 120 - 5q = 20 \]

\[ -5q = -100 \]

\[ q = 20 \]

\[ p = 120 - 5(20) = \$20 \]

\[ \Pi = 20(120 - 100) - (50 + 400) \]

\[ = 400 - 50 + 400 \]

\[ = \$50 \]

Thus, the firm's profit of \$450 would be, in this case, reduced to a \$50 loss.
For the case of an advertisement as a percentage of profit.

\[ \Pi = R(q) - C(q) - x[R(q) - C(q)] = (1-x) [R(q) - C(q)] \]  

(95)

Let \( A(q) = 10q \)

\[ \Pi = (120q - 5q^2) - (50 + 20q) - 10q = 90 - 10q = 0 \]

\[ 10q = 90 \]

\[ q = 9 \]

\[ p = \$75 \]  

(96)

\[ \Pi = (90)9 - 5(9)^2 - 50 \]

\[ = 810 - 405 - 50 \]

\[ = \$355 \]  

(97)

For the case of an advertisement as a percentage of profit.

\[ \Pi = R(q) - C(q) - x[R(q) - C(q)] \]

= (1-x) [R(q) - C(q)]  

(98)

Since \( 0 < t < 1 \), and if \( R(q) < C(q) \) which entails a loss or negative profit, advertising will incur further loss. Therefore, we can conclude that if a firm wants to improve his profit situation, his amount of advertising should be determined by his sales, not his profit.
Fig. 1. (ref. p. 2-3)

Fig. 2. (ref. p. 2-3)

Pure competitor:

\[ \frac{dP}{dQ} = 0 \Rightarrow P = \frac{Q}{MR} = 0 \]

\[ MR = p \]

Since \( \frac{dP}{dQ} < 0 \),

\[ D = MR < p \]
PART IV

GRAPHICAL ILLUSTRATIONS
Fig. 3 (ref. pp. 6)

TR

E_0 or e = 1

(e>1)

R = af + bg^2

Fig. 4 (ref. pp. 6)

\[
\frac{d(TR)}{dq} = \frac{d}{dq} (aq - bq^2) = a - 2bq = MR.
\]
Fig. 5 (ref. pp. 7 - 8)
Fig 6 (ref. p.p. 5-6)

*Unitarily elastic*

\[ e = 1 \quad \therefore - \frac{dp}{dq} = -1 \times 1 = 1 \]

*Perfectly elastic*

\[ e = +\infty \]

*Perfectly inelastic*

\[ e = -\infty \]
Fig. 9 (ref. pp. 15-18)

PRICE CONTROL BY DISCRIMINATING MONOPOLIST
The Cournot Monopoly Solution

\[ p = F(q_1 + q_2) \]

Fig. 10 (ref. pp. 25-27, pp. 45-49)

The Market-share Solution for homogeneous product

Fig. 11 (ref. pp. 52-53)
Market-Sharing Solution

Fig. 12 (a) (ref. pp. 53-56)

Firm I

Firm II
The Kinked Demand Curve

Fig. 13 (ref. pp. 57-58)

Resultant Kinked Demand Curve

Fig. 14 (ref. pp. 59)
Fig. 15 (ref. pp. 62-63)

Fig. 16 (ref. pp. 63-64)
Direct Effect of Advertising

Fig. 17 (ref. pp. 68)

Optimal Level of Advertising

Fig. 18 (ref. pp. 69)
Secondary Effect of Advertising

Fig 19 (a) (ref. pp 70)
SELECTED REFERENCES


