Spring 2015

Terraforming Mars via Aerobraking an Asteroid

Austin Powell
Carroll College, Helena, MT

Follow this and additional works at: https://scholars.carroll.edu/mathengcompsci_theses

Part of the Applied Mathematics Commons, Astrophysics and Astronomy Commons, and the Other Engineering Commons

Recommended Citation
https://scholars.carroll.edu/mathengcompsci_theses/11

This Thesis is brought to you for free and open access by the Mathematics, Engineering and Computer Science at Carroll Scholars. It has been accepted for inclusion in Mathematics, Engineering and Computer Science Undergraduate Theses by an authorized administrator of Carroll Scholars. For more information, please contact tkratz@carroll.edu.
This thesis for honors recognition has been approved for the Department of Mathematics.

Director

Date

Reader

Date

Reader
Terraforming Mars via Aerobraking an Asteroid

Austin Powell

December 1, 2015

Abstract

Terraforming Mars is a tantalizing topic for the future of humanity. It has been estimated that an increase in temperature of about four degrees Kelvin would result in a runaway greenhouse effect on Mars, raising the temperature and atmospheric pressure on Mars to habitable conditions. One of the proposed methods for terraforming Mars involves impacting an asteroid to raise the temperature by four degrees Kelvin. There has been apprehension at this method, though, due to the destructive capabilities of asteroids. Michio Kaku proposes that we use the atmosphere to slow the impacting body before collision. Modeling this proposition as an aerobrake maneuver, we explore thermal energy delivered during aerobraking as a percentage of the total energy delivered to Mars. We used a Runge-Kutta 4th order method to model the aerobraking of an asteroid from the Martian gravitational sphere of influence to different initial aerobraking altitudes. With an asteroid matching the physical aspects of 99942 Apophis (density of about $2 \text{ g cm}^{-3}$, mass of $4 \times 10^{10} \text{ kg}$), only about 46% of the energy delivered can be dissipated via aerobraking. However, a comet with a density similar to Halley’s Comet (about $0.6 \text{ g cm}^{-3}$) and a much smaller mass than 99942 Apophis (about $5 \times 10^{8} \text{ kg}$) can achieve around 56% of total energy delivered as thermal energy from aerobraking. The mass of the asteroid ultimately determines how large of a temperature increase the aerobraking body delivers. An asteroid matching 99942 Apophis can deliver the necessary energy to raise the temperature by four degrees Kelvin whereas around twenty comets matching the density of Halley’s Comet with a mass of $5 \times 10^{8} \text{ kilograms}$ would be needed to deliver the necessary energy.
1 Introduction

Since the dawn of mankind, we have been explorers. This exploration has taken us across continents, across oceans, across many barriers that were previously thought to be insurmountable. Now we have spread over the planet and the question arises: can we go further? Is it feasible to hurdle the massive obstacle that is space and thrive on a different planet? Our attention turns towards the most Earth-like of the planets: ones that have water, atmosphere, and probably are in the habitable zone of the Sun. Eventually, our attention turns to Mars as a potential planet to inhabit. However, we could not just land on Mars and begin thrive. We need to be able to breath its atmosphere, walk without a pressurized space suit, and be able to survive its temperature; in other words, Mars needs to be terraformed first.

1.1 Why Terraform Mars?

In many ways, Mars is similar to Earth. Mars has a 24.61 hour day and an axial tilt of 25.2 degrees (vs. Earth’s 23.5 degrees.) Of the planets in our solar system, Mars most closely resembles Earth in aspects important to terraforming. However, there are also striking disparities that make terraforming Mars a daunting task.

1.1.1 Martian Terraforming Profile

Mars’s terraforming profile can be summed up relatively quickly via its atmospheric pressure, atmospheric composition, and temperature. Unlike Earth, Mars does not have active volcanoes. Therefore, there is no volcanic activity to replenish the atmosphere. This leaves Mars with a surface pressure of 0.008 atmospheres. Additionally, Mars has no oxygen in its atmosphere. Thus, its atmosphere primarily consists of CO$_2$ ($\sim$ 95%). Also, the mean temperature of Mars is about $-60$°C [4]. Finally, Mars only has local magnetospheres to protect the atmosphere against the solar wind [11].

1.1.2 Potential for Terraforming

In order for humans to be able to live on Mars, its air pressure must be increased, the atmosphere must have a breathable composition, and the mean temperature of Mars needs to be raised to around 0°C [4]. These are significant undertakings to even consider but there are facets of Mars that may make this transformation possible. First, there is an enormous amount of CO$_2$ stored in the Martian ice caps and regolith. If the global temperature were raised by only a few degrees, enough CO$_2$ would sublimate to give an atmospheric pressure of 0.33 atmospheres [16]. This is comparable to the air pressure on Mount Everest (0.3 atmospheres) and is habitable [16]. In tandem with the atmospheric pressure, a temperature increase of approximately 4 degrees Kelvin would result in a runaway greenhouse effect [18]. This greenhouse effect would increase the mean Martian temperature as well as the atmospheric pressure. The habitability of Mars would last for many generations, though estimating how long the atmosphere would last without volcanic activity is hard due to the lack of a global magnetosphere [13]. We can address two of the three main problems for terraforming Mars by simply increasing the average Martian temperature. The next step in terraforming would then be increasing the supply of $O_2$ in the atmosphere and adding a buffer gas (such as nitrogen on Earth). Therefore, if we could deliver enough energy to the Martian atmosphere, then we could surmount two of the biggest problems facing the habitability of Mars.
1.2 Ethics

Along with the question of whether we can terraform Mars, lies the question should we terraform Mars. The only references we have to answer this question is our current environmental ethics: the ethics we use to describe the relationship between humanity and the Earth.

1.3 Axioms of Environmental Ethics

It is proposed that any system of environmental ethics is grounded in one of three axioms: anti-humanism, wise stewardship, and intrinsic worth [5]. Anti-humanism states that humanity does not have the right to alter its environment with its technology. This axiom is based on the principles of humanity and “original sin.” Anti-humanism would be against terraforming Mars. The second axiom, wise stewardship, says that humanity must be responsible and intelligent in using nature to benefit present and future generations. This axiom is grounded in the ideal that humanity should use the environment but must keep future generations in mind. This axiom would not be opposed to terraforming Mars through the scientific benefits and the possibility of alleviating overpopulation on Earth. The final axiom, intrinsic worth, argues that living systems have value beyond the use of humans. This view would not be opposed to terraforming Mars as long as the planet was devoid of life. [5]

1.4 Current Environmental Perspectives

Along with the axioms, we look to current environmental perspectives to see if terraforming Mars is ethical. Many perspectives involving the intrinsic value of life have different answers depending on if Mars is desolate or if it could be housing microbes. If the latter is true, then it could be considered invading another planet.

1.4.1 Anthropocentrism

Anthropocentrism is grounded in the belief that humans have a higher value than other living creatures because we are rational. Therefore, we are the sole holders of morality and ultimately decide what is right and wrong. This view has evolved over time to include future generations of humanity and supports maintaining the biosphere for the survivability of future humans. This view is not against terraforming Mars since Mars could provide valuable scientific research and a place to live [5].

1.4.2 Zoocentrism

Zoocentrism proposes that sentient animals have similar rights as humans. Therefore, it is illegitimate to exploit sentient animals for food, research, or any degrading process. Note that this does not include microorganisms as they are not sentient. This view would not be opposed to terraforming Mars because any life on Mars would likely be in the form of microbes. There would be no sentient creatures wronged [5].

1.4.3 Ecocentrism

Ecocentrism holds that the whole living world is interconnected, forming part of a whole. No species is considered inferior or superior to another. This view maintains that we must maintain the stability and integrity of the whole living world. Ecocentrism assigns intrinsic value to all living things, regardless of sentience. Therefore, if Mars were home to microbes, Ecocentrism would be against terraforming. However, in order to support terraforming, Ecocentrists would need strong evidence to suggest that Mars did not house microbes[5].
1.4.4 Cosmic Preservationism

Cosmic Preservationism assigns intrinsic value to uniqueness. This value argues that the cosmos must be preserved and should not be altered by humanity in any way. Therefore, this view wholeheartedly rejects terraforming Mars, regardless of whether it is desolate. Cosmic Preservationism would rather humanity be an observer of the universe than an invader [5].

1.5 Theoretical Methods of Terraforming

Leaving the ethical considerations aside momentarily, we also must ask how we would terraform the Martian landscape. Many methods of terraforming have been evolved to address the three main problems discussed in the Martian terraforming profile.

1.5.1 Algae and Bacteria

The largest problem with terraforming Mars lies in the composition of its atmosphere. We could not breathe the atmosphere as it is and there are no reserves of oxygen anywhere on the planet. To combat this oxygen shortage, NASA is currently testing whether any algae and bacteria could survive the harsh climate of Mars and produce oxygen by removing nitrogen from the soil. NASA has built an artificial environment on Earth to simulate the harsh surface of Mars and is currently testing the algae and bacteria [15]. On Earth, between 70 and 80 percent of the oxygen is produced by photosynthetic algae and cyanobacteria [15]. Therefore, algae and bacteria seem to be potential candidates in terraforming the Martian atmosphere.

1.5.2 Optical Mirrors

Another proposed method to terraform Mars focused on heating the planet is to use orbital reflectors. As seen in Figure 1.1, these reflectors would target small patches of Martian surface and raise the temperature of these areas. While the temperature increase would only be local, the reflector could release CO$_2$ trapped in the ice caps to trigger a runaway greenhouse effect. [18]

1.5.3 Asteroid Impact

A more dramatic and faster method of raising Mars’ average temperature would be to direct an asteroid or comet into the Martian surface. Depending on the energy of the asteroid, enough energy can be released to raise the global temperature. A change of only four degrees Kelvin would result in a runaway greenhouse effect, increase the atmospheric pressure and could raise the temperature to desired levels within a decade. [18] Along with this, there is evidence that impacts in the past “pushed Mars to a long-lasting hot runaway state.” [13] It is also possible that past impacts restarted some volcanic activity and helped maintain the hot runaway state. [9] However, there are reservations about using an asteroid since an asteroid could cause damage to the Martian surface and could be lethal to any microbes there.

In light of these reservations, theoretical physicist, Michio Kaku, proposed that “we can accurately aim the comet or meteor so that we can minimize surface damage but maximize energy transfer.” [8] Essentially, to reduce the energy released at impact, the aerobraking body must lose a large proportion of its energy to air resistance in the atmosphere. In this investigation, our model will simulate the aerobraking that Kaku discusses and determine the percentage of initial kinetic energy that can be diverted to the atmosphere through an aerobraking process and determine the proportion of the initial kinetic energy delivered at impact.
2 Model

We will simulate the aerobraking of an asteroid or comet through the Martian atmosphere. The model will take place inside the Martian sphere of influence (SOI) with an asteroid/comet initially positioned at the very edge of the SOI. The asteroid will have just enough kinetic energy to create an elliptical orbit with the apoapsis (point of highest orbit) at the SOI and periapsis (point of lowest orbit) at a defined aerobraking altitude. This simulation will show how much kinetic energy can be transferred to the atmosphere before impact.

2.1 Assumptions

- The asteroid/comet is spherical with a drag coefficient of 0.47, the approximate drag coefficient for a rough sphere.
  There are a wide variety of comets and asteroids that could be candidates for this aerobraking maneuver. This model assumes one shape (a sphere) with one drag coefficient (0.47). The method can be applied to a variety of asteroids or comets as their dimensions become known.

- The asteroid/comet begins in an elliptical orbit with apoapsis at the Martian sphere of influence and periapsis at the desired aerobraking altitude.
  For an aerobraking maneuver, the asteroid or comet cannot leave the sphere of influence of Mars. Therefore, we initially position the aerobraking body at the edge of the Martian sphere of influence so that we know it will not escape into deep space and that the Martian gravity is significant enough to form the elliptical orbit we desire.

- The asteroid/comet does not break up in the atmosphere. Asteroids and comets can have their densities vary from less than $1 \frac{g}{cm^3}$ to around $5 \frac{g}{cm^3}$.[1] The density depends on the asteroid’s composition and how porous the asteroid is. A lower density asteroid will often be porous and will
be more likely to break up during the aerobraking process than a more dense asteroid.

- Nearly all of the energy of impact goes to heat.[10]

- The force of the air resistance of the Martian atmosphere on the aerobraking body is given by Equation 1. (note: the equation is quadratic due to the speed of the aerobraking body through the atmosphere)

\[ F_{\text{drag}} = -\frac{1}{2} C \rho A v^2 \]  

(1)

In Equation 1, \( C \) is the drag coefficient, \( \rho \) is the density of the atmosphere, \( A \) is the cross-sectional area of the aerobraking body, and \( v \) is the aerobraking body’s velocity through the atmosphere.

- All kinetic energy lost during aerobraking goes to heating the atmosphere.

- The vast majority of the Martian atmosphere is comprised of CO\(_2\) [14].

- Mars has the following dimensions [17]:
  
  Radius: 3396.2 km
  Mass: 6.4174 \( \times 10^{23} \) kg
  Total mass of atmosphere: \( \sim 2.5 \times 10^{16} \) kg

- The Martian atmosphere is modeled by the following equations from NASA’s website [7]:
  
  For altitude > 7000 meters:

\[ T(a) = -23.4 - 0.00222 \cdot a \]  

\[ p(a) = 0.699 \cdot e^{-0.00099 \cdot a} \]  

(2a)  

(2b)

For altitude < 7000 meters:

\[ T(a) = -31 - 0.000998 \cdot a \]  

\[ p(a) = 0.699 \cdot e^{-0.00099 \cdot a} \]  

(3a)  

(3b)

(3c)

where \( T \) is temperature(°C), \( p \) is pressure(K-Pa) and \( a \) is altitude (m). Then, the density of the atmosphere (\( \frac{kg}{m^3} \)) is given by the following equation [7]:

\[ \rho = \frac{p}{0.1921(T + 273.1)} \]  

(4)

2.2 Parameters

- Initial aerobraking altitude (\( a \))

- Mass of asteroid/comet (\( m \))

- Radius of asteroid/comet (\( r_a \))

We will vary these parameters to find the percentage of energy transferred to the atmosphere through aerobraking. We will also see how many orbits the asteroid or comet made and how long the aerobraking process will take.
2.3 Method

The system was modeled in two dimensions using the fourth-order Runge-Kutta method with a time step of 2 seconds. This model will run until the asteroid impacts on Mars, or when the altitude of the orbiting body is less than the orbiting body’s radius. The model will output the thermal energy delivered to the atmosphere and the energy delivered on impact. From this information, we can find what percentage of total energy delivered to Mars was delivered as thermal energy during aerobraking. This will be deemed the “efficiency” of the aerobrake. The MATLAB code for this method can be found in Appendix C.

2.4 Orbital Setup

Each orbit has a total mechanical energy associated with it that is constant throughout the orbital trajectory if there is no air resistance. The total mechanical energy can be found by summing the potential and kinetic energy of the orbiting object as shown in Equation 5.

\[
\text{Total Orbital Energy} = \frac{1}{2}mv^2 - \frac{mGM}{r} \quad (5)
\]

where \( r \) is the radial distance from the center of Mars, \( M \) is the mass of Mars, \( G \) is the universal gravitation constant, and \( v \) is the speed of the asteroid or comet. Another way to find the total orbital energy using the apoapsis and periapsis is shown in Equation 6.

\[
\text{Total Orbital Energy} = \frac{mGM}{\text{apoapsis} + \text{periapsis}} \quad (6)
\]

Here, our apoapsis will be Mars’ sphere of influence and our periapsis will be the radius of Mars plus our desired aerobraking altitude. Using Equations 5 and 6, we can find the initial speed of the orbiting body needed to orbit from the Martian SOI to the desired aerobraking altitude.

2.5 Energy Delivered

The energy delivered to the atmosphere during aerobraking can be found by calculating the difference in the initial orbital mechanical energy and the final orbital mechanical energy just before impact using Equation 5. The only agent causing a change in the orbital mechanical energy is the drag force on the orbiting body given by Equation 1. Therefore, any energy lost from the initial orbital mechanical energy to the final orbital mechanical energy is delivered as thermal energy to the atmosphere.

The energy delivered on impact can be found via the kinetic energy at impact. The meteor, at this point, will still have potential energy. Therefore, the energy delivered on impact will only be the kinetic energy at the time of the collision.

2.6 Temperature Increase

After finding the energy delivered from aerobraking and on impact, the temperature rise is found with the specific heat of CO\textsubscript{2} at the mean Mars temperature (-60° C) and the mass of the Martian atmosphere. The specific heat(\( C_p \)) of CO\textsubscript{2} at -60 °C is around 0.74 \( \text{J kg}^{-1} \text{K}^{-1} \). The temperature increase, in Kelvin, is then given by Equation 7.

\[
\Delta T = \frac{\text{Energy}}{C_p \cdot m} \quad (7)
\]

Specifically for this case, the temperature increase on Mars given some amount of delivered energy is given by Equation 8.

\[
\Delta T = \frac{\text{Energy}}{0.74 \cdot 2.5 \times 10^{16}} \quad (8)
\]
2.7 Verification of Method

A numerical model must be verified before it is used to gather results. To verify a numerical model, we must check that it converges so that a change in step size will not yield different results. We must also ensure that the model conserves quantities that are conserved in the real world. In this model, we will check that we conserve energy throughout the simulation without air resistance. Finally, to verify a model we must make sure that the simulation behaves as expected in certain situations. In our model, we will look at the consistency of an orbit without air resistance, the expected rise in temperature vs. the simulated rise in temperature, and the expected behavior of an aerobraking body.

2.7.1 Convergence

If the model is accurate, then altering the time step should not significantly change the results. To check that our model converged, we reduced the time step by half (to 1 second) and ran the simulation again. The efficiency, time before impact, and the total temperature increase had maximum changes of 0.2%, 0.07%, and 0.002% respectively. Therefore, the model had converged to beyond the precision we were using.

2.7.2 Energy

The total mechanical energy in an orbit is conserved. We must check that the model conserves mechanical energy along a single orbital trajectory. To do this, the simulation was run with no air resistance at a time period longer than observed in data collecting (150 days). We would expect that the asteroid or comet would continue on the same orbit indefinitely, with no change to the orbital energy. The range of the energy was divided by the average energy to give a relative error as shown in Equation 9.

\[
\text{Relative Error} = \frac{\max(\text{Energy}) - \min(\text{Energy})}{\text{AverageEnergy}} \quad (9)
\]

This resulted in a relative error on the order of \( -3 \times 10^{-11} \). This is very acceptable error for the model.

2.7.3 Orbital Consistency

Again, without air resistance, we would expect that the asteroid or comet would continue on the same orbit indefinitely, with the same apoapsis and periapsis. After 150 days, the changes in the apoapsis and the periapsis were observed. The orbital trajectory for this verification can be seen in Figure 2.1. The variation of the apoapsis was \( 4.15 \times 10^{-4} \) meters and the periapsis was missed by 0.0740 meters. Given the magnitude of distances involved, these are acceptable errors.

2.7.4 Expected Rise in Temperature

We can find the expected rise in temperature from the initial orbital mechanical energy and the final potential energy of the meteor. The aerobraking body will start with a given amount of energy according to Equation 6 based on its apoapsis and periapsis. The apoapsis, in this case, is the Martian SOI and the periapsis is the radius of Mars. Note that the initial aerobraking altitude is negligible when calculating the expected temperature increase due to the difference in magnitudes of the SOI and the initial aerobraking altitude \((5.67 \times 10^8 \text{ vs.} \ 1 \times 10^3)\). After impact, the aerobraking body will be at rest on the surface of Mars, its only mechanical energy being the potential energy from Mars’ gravitational field since we treat Mars’ mass as being at the center of the planet. Therefore, we know the initial mechanical energy and final mechanical energy of the aerobraking body. The difference in energy from the initial orbit to impact is the energy that the meteor imparted to the Martian atmosphere. From the energy delivered,
Figure 2.1: plot of asteroid orbit over 150 days (density=1.93 \(\text{g cm}^{-3}\), aerobraking altitude of 10000 m) note that Mars is represented by a single circle that does not actually represent its size.

we can find the predicted temperature rise for a given aerobraking body. We would expect that across all simulations involving a particular aerobraking body, the rise in temperature matches this calculation and is constant. Note that the initial aerobraking altitude for a particular aerobraking body was changed between simulations. The results are displayed in Table 1. The error in these values is very acceptable.

<table>
<thead>
<tr>
<th>Mass of Aerobraking Body</th>
<th>Predicted Temp Rise</th>
<th>Average Temp Rise</th>
<th>Standard Dev</th>
<th># of Simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 (\times 10^{10}) kg</td>
<td>27.1067 °C</td>
<td>27.1055 °C</td>
<td>0.000127301 °C</td>
<td>23</td>
</tr>
<tr>
<td>5 (\times 10^{8}) kg</td>
<td>0.3388 °C</td>
<td>0.3388 °C</td>
<td>4.6347 (\times 10^{-5}) °C</td>
<td>70</td>
</tr>
</tbody>
</table>

Table 1: The expected increase in temperature for both of the masses of aerobraking bodies used in data collection along with the average temperature increase over all simulations and the standard deviation in these temperature increases. Notice that the predicted temperature rise matches the average temperature rise throughout all simulations and that the standard deviation is relatively small.

2.7.5 Expected Behavior

With air resistance, there are several aspects of the orbital trajectory that we should expect. First, the total mechanical energy is conserved until the asteroid enters a significant density of the Martian atmosphere. At this point, orbital energy is lost due to air resistance until the asteroid leaves the atmosphere. We expect to see the total mechanical energy “stair-stepping” down over time as the asteroid makes passes through the atmosphere. We also expect to see the orbit decay in steps as well, with the apoapsis decreasing every time the asteroid passes through the atmosphere. Since the aerobraking body is in a bound orbit, the energy should also be negative throughout the entire maneuver. These effects can be seen in Figure 2.2.
Figure 2.2: Energy and trajectory plots for asteroid with density 2.23 \( \text{g cm}^{-3} \) and aerobraking altitude of 1000 meters

3 Results

3.1 Benchmark Efficiency

The “optimal” efficiency of our aerobraking is technically near 100%. An efficiency of near 100% would occur if the meteor was slowed to its terminal velocity before impact. However, a more useful benchmark is defined when our meteor has a circular orbit just above the Martian surface. If there were no air resistance, right above the surface of Mars is the lowest that the orbit could be without the meteor impacting the ground. The mechanical energy of the lowest orbit can be calculated using Equations 5 and 6, giving a potential efficiency of 49.71%.

It is possible for a meteor to have an efficiency higher than our benchmark efficiency. If, during its final descent, the meteor is slowed by air resistance enough, it will actually lower its orbital mechanical energy beyond this point. Lowering the mechanical energy beyond our benchmark can only happen during the final pass through the atmosphere and with relatively low-density meteors.

3.2 Initial Aerobraking Altitude and Number of Complete Orbits

3.2.1 High Density

There are many asteroids which are composed of rocks and metals. These asteroids tend to have a higher density than comets. The density of these bodies is around 2-3 \( \text{g cm}^{-3} \). Although it is difficult to measure the physical aspects of most asteroids, near-earth asteroids can be studied in more detail. One such asteroid is 99942 Apophis, a near-earth asteroid that was measured to have a mass of about \( 4 \times 10^{10} \) kg and a radius of about 162.5 meters [3]. This asteroid was nearly spherical and thus, somewhat conforms to our assumptions. For this model, we will use an asteroid with a density of 2.23 \( \text{g cm}^{-3} \) to match 99942 Apophis.

Figure 3.1 shows the general trend that increasing aerobraking altitude increases the efficiency of the aerobraking maneuver. As the initial aerobraking altitude in increased, the asteroid passes through less dense atmosphere, reducing its drag. Thus, the asteroid’s orbital mechanical energy is reduced by smaller increments, allowing more energy to be dispersed in the atmosphere before the asteroid’s ultimate descent to the Martian surface. However, notice that the aerobrake efficiency does not exceed our analytic benchmark that we defined earlier. The drag does not lower the mechanical energy of this object’s orbit enough to exceed our benchmark efficiency. This is because the higher-density and increased mass of the

---

11
object makes it less susceptible to air resistance.

Figure 3.2 shows the efficiency of the aerobrake given the number of complete orbits that the aerobraking body made. This shows a similar trend to the aerobraking altitude. As we increase the aerobraking altitude, the number of complete orbits made by the aerobraking object increases as well. Figure 3.2 shows the general trend that if the number of complete orbits is increased, the efficiency increases.

3.2.2 Low Density

A comet, such as Halley’s Comet, has a density of around 0.6 \( \text{g cm}^{-3} \) [12]. Comets were proposed as a method for terraforming by Michio Kaku because of their potential to be slowed by the atmosphere and their potential to break up in the atmosphere, further reducing the impact [8]. Figure 3.3 shows the efficiency of aerobraking a body with the density close to 0.6 \( \text{g cm}^{-3} \) at different initial periapsis altitudes. The data from all simulations for a low density body is given in Appendix A.

First, with a density of 0.596 \( \text{g cm}^{-3} \), the meteor can achieve an efficiency higher than our analytic benchmark at certain periapsis altitudes as shown in Figure 3.3. For instance, at an initial aerobraking altitude of 10,000 meters, the efficiency of the aerobraking is approximately 52%. This means that 52%
Figure 3.3: Efficiency of aerobraking vs altitude (density of $0.596 \text{ g cm}^{-3}$)

Figure 3.4: The final orbit of a meteor with density of $0.596 \text{ g cm}^{-3}$ with an initial aerobraking altitude of 10000 meters and aerobraking efficiency of 52%

of the thermal energy delivered to Mars was through aerobraking. If we examine the crash trajectory of the 10,000 meter aerobraking altitude shown in Figure 3.4, we can see the behavior that causes such high efficiency. For high efficiency, during the second to last pass through the atmosphere, the aerobraking body must lose enough energy to just barely make it out of the atmosphere. This means that the orbiting body is in a very low orbit, with low orbital mechanical energy. Then, when the meteor makes its final pass through the atmosphere, enough mechanical energy is lost to beat the benchmark efficiency.

There are also “jumps” in the efficiency that can be explained by the number of orbits as shown in Figure 3.5. When the aerobraking altitude is increased to the point that the orbiting body makes one more complete orbit, there is a spike in efficiency. This is due to the behavior shown in Figure 3.4. When the aerobraking body barely escapes the atmosphere, it can make one more low orbit before
Figure 3.5: Efficiency of aerobraking and number of complete orbits made vs altitude (density of 0.596 (g/cm³))

Figure 3.6: The final orbit of a meteor with density of 0.596 g/cm³ with an initial aerobraking altitude of 3250 meters and aerobraking efficiency of 30% impact. However, increasing the initial aerobraking altitude beyond this decreases efficiency until a new orbit is made. When the initial aerobraking altitude is increased past a new orbit, the crash trajectory is closer to that shown in Figure 3.6 where the orbit is not brought closer to the surface of Mars before the final pass through the atmosphere. Notice that the efficiency of aerobraking of a higher density asteroid does not have the “jumps” that Figure 3.5 shows. This is because the high density and more massive asteroid is not as affected by drag and makes many more complete orbits with a small increase in initial aerobraking altitude.
Figure 3.7: Efficiency vs the number of complete orbits made by the aerobraking body (density = 0.596 g/cm³)

We can also look at the different efficiency values for each orbit as shown in Figure 3.7. We can see that the range of potential efficiencies increases as we increase the number of complete orbits. We can also see that the average efficiency increases as the number of complete orbits increases as was the case with the high density object.

3.3 Time

To determine how long the aerobraking maneuver takes, we will look at the time it takes for the asteroid to impact given our defined parameters. Since the higher density object is less affected by drag, we would expect the higher density object to take longer to impact than the lower density object.

3.3.1 High Density

The time until impact for the high-density aerobraking object given an initial aerobraking altitude is shown in Figure 3.8. The high-density object makes many more complete orbits given an increase in initial aerobraking altitude (around 6 more complete orbits for a 500 meter increase in aerobraking altitude). Figure 3.8 shows the general relationship between initial aerobraking altitude and the time until impact. The time until impact steadily increases as we would expect. We would also expect the time until impact to go to infinity as our initial aerobraking altitude increased beyond the influence of the Martian atmosphere.

3.3.2 Low Density

The time until impact for the low-density aerobraking object and number of complete orbits made given an initial aerobraking altitude is shown in Figure 3.9. From Figure 3.9, we can see that the time it takes to impact increases with aerobraking altitude. Also, if we increase the initial aerobraking altitude just enough so that the low-density object makes one more complete orbit, then there is a larger increase in the amount of time until impact.
4 Sensitivity

Sensitivity refers to the effect varying parameters has on our results. Parameters in the real world are rarely known exactly and can often vary over time. Sensitivity of a parameter is then how much our answer changes in relation to a change in that parameter, specifically; it is the ratio of the percent change in our answer over the percent change in that parameter. If we change a single parameter by 10%, and our answer changes by less than 10% (a sensitivity less than 1), then that parameter is not considered “sensitive.” However, if we perturb a parameter by 10% and our answer changes by more than 10% (a sensitivity greater than 1), then that parameter is considered “sensitive.” Sensitive parameters are parameters that should be measured as accurately as possible before relying on a model for predictions.

The sensitivity of parameters with the largest uncertainty are shown in Table 2.

The most estimated and difficult-to-know parameter in our model is the drag coefficient of the aer-
<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\Delta$ Parameter</th>
<th>$\Delta$ Efficiency</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drag Coefficient</td>
<td>0.047</td>
<td>5.61%</td>
<td>1.16</td>
</tr>
<tr>
<td>Mars SOI</td>
<td>$5.76 \times 10^7$ kg</td>
<td>0.1 %</td>
<td>0.02</td>
</tr>
<tr>
<td>Mass of Martian Atmosphere</td>
<td>$2.5 \times 10^{15}$ kg</td>
<td>0.01 %</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Table 2: The sensitivity of estimated parameters in the model. Note that this was done with a density of $0.69 \frac{g}{cm^3}$ with a 6,750 m initial aerobraking altitude.

The sensitivity of estimated parameters in the model. Note that this was done with a density of $0.69 \frac{g}{cm^3}$ with a 6,750 m initial aerobraking altitude.

Aerobraking body that we are using. Asteroids and comets vary greatly in size and shape. Therefore, the drag coefficient cannot be known until a specific asteroid or comet has been chosen for the aerobraking. The drag coefficient is the only parameter that is considered sensitive in this model. Therefore, the drag coefficient should be carefully measured for any object chosen for this aerobraking maneuver. Note that the calculations for sensitivity were done immediately after a new complete orbit was made (five complete orbits to six complete orbits) to maximize the potential change that a perturbation of each parameter could cause. If the perturbation of the parameter does not result in a different number of complete orbits before impact, no parameters are sensitive.

5 Conclusion

Impacting an asteroid on Mars is fraught with ethical discussion. Michio Kaku claims that we can use the atmosphere of Mars to soften the impact of a meteor. From our fourth-order Runge Kutta method modeling an aerobraking procedure around Mars, we determined the expected “efficiency” of such a maneuver.

A rocky asteroid that matches the dimensions of 99942 Apophis delivers much more energy than we need but reveals the general trends of the efficiency of aerobraking vs the aerobraking altitude. A higher initial aerobraking altitude will result in a higher efficiency to a point. Too high of an initial aerobraking altitude will result in the aerobraking body avoiding the atmosphere and orbiting indefinitely. With a sufficiently high initial periapsis, high-density, rocky asteroids can be expected to achieve up to around 46% efficiency. That is, 46% of the total energy that the meteor delivered to Mars could be delivered in the form of thermal energy while aerobraking. The rest was delivered on impact. Aerobraking objects with high density and mass do not exceed our analytic benchmark efficiency of 49.71%.

With a lower mass and a lower density object, the benchmark can be surpassed and the aerobraking behavior necessary to surpass the benchmark can be seen. With an aerobraking object whose density closely resembles Halley’s Comet, around 56% of total energy delivered was imparted to the atmosphere as thermal energy during aerobraking. The low-mass, low-density object also revealed the relationship between the number of complete orbits and efficiency as seen in Figure 3.5. In order to achieve high efficiency as we have defined it, the aerobraking object must lose just enough mechanical energy during its penultimate pass through the Martian atmosphere to still barely escape the atmosphere as seen in Figure 3.4. The mechanical energy at this point has been lowered enough that the object’s orbit is close to the Martian surface. Then, as the object makes its final pass through the atmosphere, the mechanical energy is reduced enough that we beat the benchmark efficiency of 49.71 %. Therefore, if we look at efficiency vs orbits, when we increase the initial aerobraking altitude just enough to result in the comet making another complete orbit, we have local peaks in efficiency. However, it would take many more low-mass objects to get the desired temperature increase. With the low-density object we modeled, it would take approximately 15 such objects to put Mars in a runaway greenhouse state since an object with a mass of $5 \times 10^8$ kg delivers only enough energy to raise the temperature by 0.34 Kelvin. Finally,
low-density objects are likely to break up in the atmosphere, further reducing the impact and increasing the thermal energy delivered to the atmosphere.

For future studies, it would be important to find the impulse necessary to move a potential asteroid or meteor into a, closed, elliptical orbit around Mars for the purpose of aerobraking. In tandem with finding the impulse is the mechanism in which the impulse is imparted to the orbiting body. It would also be useful to explore the potential and probability of meteors exploding in the Martian atmosphere during the final descent as this is a major point discussed by Michio Kaku.

We can disperse the total energy that would be delivered on impact by anywhere from 45% to 56% to the Martian atmosphere depending on the mass and density of the object used in aerobraking. It is up for debate whether this is a “soft” enough landing for those with ethical and practical concerns. Regardless, an aerobraking maneuver of an asteroid could result in a more efficient transfer of energy to Mars and prove to be a useful method of terraforming. As explorers, our near future is filled with the new possibilities science offers us and one of these possibilities could involve inhabiting a new planet.
References


<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
</tr>
</tbody>
</table>

Appendix A: Simulation Data for Low Density
<p>| | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>22</td>
<td>22</td>
<td>22</td>
<td>22</td>
<td>22</td>
<td>22</td>
<td>22</td>
<td>22</td>
<td>22</td>
<td>22</td>
<td>22</td>
</tr>
<tr>
<td>24</td>
<td>24</td>
<td>24</td>
<td>24</td>
<td>24</td>
<td>24</td>
<td>24</td>
<td>24</td>
<td>24</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>26</td>
<td>26</td>
<td>26</td>
<td>26</td>
<td>26</td>
<td>26</td>
<td>26</td>
<td>26</td>
<td>26</td>
<td>26</td>
<td>26</td>
</tr>
<tr>
<td>28</td>
<td>28</td>
<td>28</td>
<td>28</td>
<td>28</td>
<td>28</td>
<td>28</td>
<td>28</td>
<td>28</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>32</td>
<td>32</td>
<td>32</td>
<td>32</td>
<td>32</td>
<td>32</td>
<td>32</td>
<td>32</td>
<td>32</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>34</td>
<td>34</td>
<td>34</td>
<td>34</td>
<td>34</td>
<td>34</td>
<td>34</td>
<td>34</td>
<td>34</td>
<td>34</td>
<td>34</td>
</tr>
<tr>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>38</td>
<td>38</td>
<td>38</td>
<td>38</td>
<td>38</td>
<td>38</td>
<td>38</td>
<td>38</td>
<td>38</td>
<td>38</td>
<td>38</td>
</tr>
<tr>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>42</td>
<td>42</td>
<td>42</td>
<td>42</td>
<td>42</td>
<td>42</td>
<td>42</td>
<td>42</td>
<td>42</td>
<td>42</td>
<td>42</td>
</tr>
<tr>
<td>44</td>
<td>44</td>
<td>44</td>
<td>44</td>
<td>44</td>
<td>44</td>
<td>44</td>
<td>44</td>
<td>44</td>
<td>44</td>
<td>44</td>
</tr>
<tr>
<td>46</td>
<td>46</td>
<td>46</td>
<td>46</td>
<td>46</td>
<td>46</td>
<td>46</td>
<td>46</td>
<td>46</td>
<td>46</td>
<td>46</td>
</tr>
<tr>
<td>48</td>
<td>48</td>
<td>48</td>
<td>48</td>
<td>48</td>
<td>48</td>
<td>48</td>
<td>48</td>
<td>48</td>
<td>48</td>
<td>48</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>52</td>
<td>52</td>
<td>52</td>
<td>52</td>
<td>52</td>
<td>52</td>
<td>52</td>
<td>52</td>
<td>52</td>
<td>52</td>
<td>52</td>
</tr>
<tr>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
</tr>
<tr>
<td>56</td>
<td>56</td>
<td>56</td>
<td>56</td>
<td>56</td>
<td>56</td>
<td>56</td>
<td>56</td>
<td>56</td>
<td>56</td>
<td>56</td>
</tr>
<tr>
<td>58</td>
<td>58</td>
<td>58</td>
<td>58</td>
<td>58</td>
<td>58</td>
<td>58</td>
<td>58</td>
<td>58</td>
<td>58</td>
<td>58</td>
</tr>
<tr>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>64</td>
<td>64</td>
<td>64</td>
<td>64</td>
<td>64</td>
<td>64</td>
<td>64</td>
<td>64</td>
<td>64</td>
<td>64</td>
<td>64</td>
</tr>
<tr>
<td>66</td>
<td>66</td>
<td>66</td>
<td>66</td>
<td>66</td>
<td>66</td>
<td>66</td>
<td>66</td>
<td>66</td>
<td>66</td>
<td>66</td>
</tr>
<tr>
<td>68</td>
<td>68</td>
<td>68</td>
<td>68</td>
<td>68</td>
<td>68</td>
<td>68</td>
<td>68</td>
<td>68</td>
<td>68</td>
<td>68</td>
</tr>
<tr>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>72</td>
<td>72</td>
<td>72</td>
<td>72</td>
<td>72</td>
<td>72</td>
<td>72</td>
<td>72</td>
<td>72</td>
<td>72</td>
<td>72</td>
</tr>
<tr>
<td>74</td>
<td>74</td>
<td>74</td>
<td>74</td>
<td>74</td>
<td>74</td>
<td>74</td>
<td>74</td>
<td>74</td>
<td>74</td>
<td>74</td>
</tr>
<tr>
<td>76</td>
<td>76</td>
<td>76</td>
<td>76</td>
<td>76</td>
<td>76</td>
<td>76</td>
<td>76</td>
<td>76</td>
<td>76</td>
<td>76</td>
</tr>
<tr>
<td>78</td>
<td>78</td>
<td>78</td>
<td>78</td>
<td>78</td>
<td>78</td>
<td>78</td>
<td>78</td>
<td>78</td>
<td>78</td>
<td>78</td>
</tr>
<tr>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>82</td>
<td>82</td>
<td>82</td>
<td>82</td>
<td>82</td>
<td>82</td>
<td>82</td>
<td>82</td>
<td>82</td>
<td>82</td>
<td>82</td>
</tr>
<tr>
<td>84</td>
<td>84</td>
<td>84</td>
<td>84</td>
<td>84</td>
<td>84</td>
<td>84</td>
<td>84</td>
<td>84</td>
<td>84</td>
<td>84</td>
</tr>
<tr>
<td>86</td>
<td>86</td>
<td>86</td>
<td>86</td>
<td>86</td>
<td>86</td>
<td>86</td>
<td>86</td>
<td>86</td>
<td>86</td>
<td>86</td>
</tr>
<tr>
<td>88</td>
<td>88</td>
<td>88</td>
<td>88</td>
<td>88</td>
<td>88</td>
<td>88</td>
<td>88</td>
<td>88</td>
<td>88</td>
<td>88</td>
</tr>
<tr>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>92</td>
<td>92</td>
<td>92</td>
<td>92</td>
<td>92</td>
<td>92</td>
<td>92</td>
<td>92</td>
<td>92</td>
<td>92</td>
<td>92</td>
</tr>
<tr>
<td>94</td>
<td>94</td>
<td>94</td>
<td>94</td>
<td>94</td>
<td>94</td>
<td>94</td>
<td>94</td>
<td>94</td>
<td>94</td>
<td>94</td>
</tr>
<tr>
<td>96</td>
<td>96</td>
<td>96</td>
<td>96</td>
<td>96</td>
<td>96</td>
<td>96</td>
<td>96</td>
<td>96</td>
<td>96</td>
<td>96</td>
</tr>
<tr>
<td>98</td>
<td>98</td>
<td>98</td>
<td>98</td>
<td>98</td>
<td>98</td>
<td>98</td>
<td>98</td>
<td>98</td>
<td>98</td>
<td>98</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

Legend:
- Column 1: Number of Tributaries
- Column 2: Number of Oases
- Column 3: Number of Reserves
- Column 4: Number of Cites
- Column 5: Number of Farms
- Column 6: Number of Schools
- Column 7: Number of Hospitals
- Column 8: Number of Universities
- Column 9: Number of Libraries
- Column 10: Number of Museums
- Column 11: Number of Zoos
- Column 12: Number of Parks

Appendix B: Simulation Data for High Density
%Austin Powell Thesis Project
%Last Modified 9/30/2015

clear space
clear all;
clc;
clf;

%Potential Asteroids
%99942 Apophis
%4*10^10 kg with 325m diameter
%near-earth caused scare for 2029 impact

%varying parameters
aeroalt=3250; %desired aerobraking altitude
dragco=.47;%drag coefficient of asteroid .47=sphere (Maybe Reynolds Function)
ma=5e8; %Mass of Asteroid in kg castella=.0005*10^15
ra=58.5; %radius of asteroid in m
density_of_asteroid=ma*1000/(4/3*pi*(ra*100)^3) %g/cm^-3

%system parameters
dt=2;%seconds
airswitch = 1;%1=air on; 0=air off
totaltime=3600*24*150; %second

%parameters
SOI=.576e9; %Mars SOI in meters
mM=.64174*10^-24; %Mass of Mars in kg
rM=3396.2e3; %radius of Mars in m
G=6.67384e-11; %universal gravitation constant m^-3/kg/s^-2

%crash collection check
crash=0;
KEcrash=0;

%Number of Orbits
cross=0;

%initial kinetic energy calculation
Emech=-ma*G*mM/(SOI+rM+aeroalt);
KE=Emech+ma*G*mM/(SOI);
vinitial=sqrt(2*KE/ma);

%mechanical energy calculation at surface of Mars
Emechpot = -ma*G*mM/(rM+rM);
KEPotential = Emechpot + ma*G*mM/(rM);
Potential_Thermal = Emech - Emechpot;
Potential = Potential_Thermal / (Potential_Thermal + KEPotential);

% preallocation
x = zeros(1, totaltime / dt + 1);
y = zeros(1, totaltime / dt + 1);
vx = zeros(1, totaltime / dt + 1);
vy = zeros(1, totaltime / dt + 1);
alt = zeros(1, totaltime / dt + 1);
MechEnergy = zeros(1, totaltime / dt + 1);

% Initial Conditions
x(1) = -SOI;  % Mars SOI in meters
y(1) = 0;  % Mars SOI in meters
vx(1) = 0;  % Initial velocity in m/s
vy(1) = vinitial;  % Initial velocity in m/s
r = sqrt(x(1)^2 + y(1)^2);  % Initial r value
MechEnergy(1) = 0.5*ma*(vx(1)^2 + vy(1)^2) - ma*G*mM/(r);  % Initial energy

% Main Loop
for n = 1:totaltime / dt
    % 4th order Runge-Kutta
    % K1
    r = sqrt(x(n)^2 + y(n)^2);
    Fgrav = G*mM*ma/r^2;
    Fx = -Fgrav*(x(n))/r;
    Fy = -Fgrav*(y(n))/r;
    ax = Fx/ma;
    ay = Fy/ma;
    alt(n) = (r - rM);
    if (alt(n) < 7000)
        pressure = 0.699*exp(-0.00009*alt(n));
        T = -31 - 0.00098*alt(n);
    else
        pressure = 0.699*exp(-0.00009*alt(n));
        T = -23.4 - 0.00222*alt(n);
    end
    density = pressure / (0.1921*(T+273.1));
    airres = 1/2*density*(vx(n)^2 + vy(n)^2)*dragco*pi*ra^2*airswitch;
    airresx = -sign(vx(n))*airres*abs(vx(n))/sqrt(vx(n)^2 + vy(n)^2);
    airresy = -sign(vy(n))*airres*abs(vy(n))/sqrt(vx(n)^2 + vy(n)^2);
    airx = airresx/ma;
    airy = airresy/ma;
    k1x = vx(n);
k1y = vy(n);
k1vx = ax + airx;
k1vy = ay + airy;

%K2
xext = x(n)+k1x*dt/2;
yext = y(n)+k1y*dt/2;
vxext = vx(n)+k1vx*dt/2;
vyext = vy(n)+k1vy*dt/2;
r=sqrt((xext)^2+(yext)^2);
Fgrav=G*mM*ma/r^2;
Fx=-Fgrav*(xext)/r;
FY=-Fgrav*(yext)/r;
ax=Fx/ma;
ay=Fy/ma;
alt(n)=(r-rM);
if(alt(n)<7000)
    pressure=.699*exp(-.00009*alt(n));
    T=-31-.000998*alt(n);
else
    pressure=.699*exp(-.00009*alt(n));
    T=-23.4-.00222*alt(n);
end
density=pressure/(.1921*(T+273.1));
airres=1/2*density*((vxext)^2+(vyext)^2)*dragco*pi*ra^2*airswitch;
airresx=-sign(vxext)*airres*abs(vxext)/sqrt(vxext^2+vyext^2);
airresy=-sign(vyext)*airres*abs(vyext)/sqrt(vxext^2+vyext^2);
airx=airresx/ma;
airy=airresy/ma;
k2x = vxext;
k2y = vyext;
k2vx = ax + airx;
k2vy = ay + airy;

%K3
xext = x(n)+k2x*dt/2;
yext = y(n)+k2y*dt/2;
vxext = vx(n)+k2vx*dt/2;
vyext = vy(n)+k2vy*dt/2;
r=sqrt((xext)^2+(yext)^2);
Fgrav=G*mM*ma/r^2;
Fx=-Fgrav*(xext)/r;
FY=-Fgrav*(yext)/r;
ax=Fx/ma;
ay=Fy/ma;
alt(n)=(r-rM);
if(alt(n)<7000)
pressure=.699*exp(-.00009*alt(n));
T=-31-.000998*alt(n);
else
pressure=.699*exp(-.00009*alt(n));
T=-23.4-.00222*alt(n);
end
density=pressure/(.1921*(T+273.1));
airres=1/2*density*((vxext)^2+(vyext)^2)*dragco*pi*ra^2*airswitch;
airresx=-sign(vxext)*airres*abs(vxext)/sqrt(vxext^2+vyext^2);
airresy=-sign(vyext)*airres*abs(vyext)/sqrt(vxext^2+vyext^2);
airx=airresx/ma;
airy=airresy/ma;
k3x = vxext;
k3y = vyext;
k3vx = ax + airx;
k3vy = ay + airy;

%K4
xext = x(n)+k3x*dt;
yext = y(n)+k3y*dt;
vxext = vx(n)+k3vx*dt;
vyext = vy(n)+k3vy*dt;
r=sqrt((xext)^2+(yext)^2);
Fgrav=G*mM*ma/r^2;
Fx=-Fgrav*(xext)/r;
Fy=-Fgrav*(yext)/r;
ax=Fx/ma;
ay=Fy/ma;
alt(n)=(r-rM);
if(alt(n)<7000)
    pressure=.699*exp(-.00009*alt(n));
    T=-31-.000998*alt(n);
else
    pressure=.699*exp(-.00009*alt(n));
    T=-23.4-.00222*alt(n);
end
density=pressure/(.1921*(T+273.1));
airres=1/2*density*((vxext)^2+(vyext)^2)*dragco*pi*ra^2*airswitch;
airresx=-sign(vxext)*airres*abs(vxext)/sqrt(vxext^2+vyext^2);
airresy=-sign(vyext)*airres*abs(vyext)/sqrt(vxext^2+vyext^2);
airx=airresx/ma;
airy=airresy/ma;
k4x = vxext;
k4y = vyext;
k4vx = ax + airx;
k4vy = ay + airy;
%RK4 weighted calculations
\[ x(n+1) = x(n) + \frac{dt}{6}*(k1x + 2*k2x + 2*k3x + k4x); \]
\[ y(n+1) = y(n) + \frac{dt}{6}*(k1y + 2*k2y + 2*k3y + k4y); \]
\[ vx(n+1) = vx(n) + \frac{dt}{6}*(k1vx + 2*k2vx + 2*k3vx + k4vx); \]
\[ vy(n+1) = vy(n) + \frac{dt}{6}*(k1vy + 2*k2vy + 2*k3vy + k4vy); \]

%Mechanical Energy
\[ r = \sqrt{x(n+1)^2+y(n+1)^2}; \]
\[ alt(n+1) = r - rM; \]
\[ MechEnergy(n+1) = 0.5*ma*(vx(n+1)^2+vy(n+1)^2) - ma*G*mM/(r); \]

%Crash
if(alt(n)<ra && crash == 0)
    KEcrash = 0.5*ma*(vx(n)^2+vy(n)^2);
    crash=1;
    break
end

%Check if asteroid crossed the x-axis
if(sign(y(n)) \neq sign(y(n+1)))
    cross=cross+1;
end

%close loop
end

%final crash time
crashtime=(n-1)*dt;

%Number of Orbits
orbits=cross/2;

%trajectory plot
plot(x(:),y(:))
xlabel('x(m)')
ylabel('y(m)')
title('Trajectory','fontsize',15)

%plot planet
hold on
plot(0,0,'ro')
legend('Orbit','Mars')

%minimum altitude
if(crash==0)
    minalt = min(alt)
end
% final mechanical energy
MechFinal=.5*ma*(vx(n)^2+vy(n)^2)-ma*G*mM/(r);

% thermal energy
ThermalE = Emech-MechFinal;

% mcat
TempRaiseAtmo = ThermalE/(2.5e16*.74)
TempRaiseSurf=KEcrash/(2.5e16*.74);

% crash
if(crash == 1)
    display('Crash!')
    display(TempRaiseSurf,'Potential temp raise from impact')
    display(crashtime/3600/24,'Crash time (days)')
    display(orbits, 'Number of Orbits')
end

% Mechanical Energy Plot
figure
plot(t,MechEnergy)
xlim([0,crashtime])
xlabel('Time(s)')
ylabel('Energy(J)')
title('Energy','fontsize',15)

% End orbits
figure
plot(x(n-3500:n),y(n-3500:n))
hold on
ang=0:0.01:2*pi;
xc=rM*cos(ang);
yc=rM*sin(ang);
plot(xc,yc,'r:');
title('Crash Trajectory','Fontsize',14);
xlabel('x(m)')
ylabel('y(m)')
legend('Orbit','Mars')

if(airswitch==0)
    error=(max(MechEnergy)-min(MechEnergy))/mean(MechEnergy)
end