SET

Carroll College

Question 1. If another attribute was added to the game, how many sets would be possible?

$$(3\ colors) * (3\ fill\ options) * (3\ number\ of\ shapes) * (3\ shapes) * (3\ types\ of\ new\ attribute) = 3^5 = 243$$

243 – (2 cards chosen) = 241, $\frac{1}{241}$ *chance of finding a set.*

- Including a new attribute in our game (i.e. background color), increases the number of cards needed to play from 3^4 or 81 to 3^5 or 243 cards.
- For the algorithm to execute with this change, another line would need to be added to the second function, like *"check_bg_color"*, while still checking the same amount of cards.
- After finding card A and card B, the odds of finding a set with card C is now $\frac{1}{241}$.
- By subtracting the number of cards already chosen (in this case, 2), we can conclude that there is only one card left in the deck to make a set with the two chosen cards.

What if there was another shape, color, fill, or number of shapes added to the game?

$$(4\ colors) * (4\ fill\ options) * (4\ number\ of\ shapes) * (4\ shapes) = 4^4 = 256$$

256 – (3 cards chosen) = 253 , $\frac{1}{253}$ *chance of finding a set.*

- Adding another attribute changes the number of cards need to play from 3^4 or 81 to 4^4 or 256.
- This means that instead of making a set of three cards, a set now needs to be made with four cards.
- If you find card A, B, and C the odds of finding a set with card D is now $\frac{1}{253}$.
- Similar to the solution above, there is only one card that can satisfy the chosen cards in order to form a set.

Citations:

https://www.amazon.com/SET-Family-Game-Visual-Perception/dp/B00000IV34

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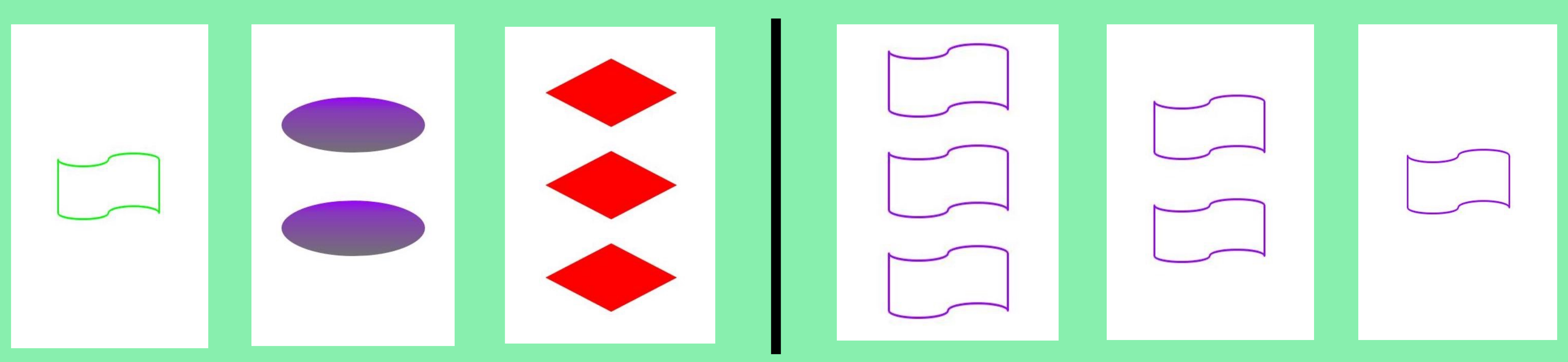
MA-328 Modern Apps of Discrete Mathematics

What is *Set?*

- Set is a logic game comprised of 81 different cards
- Each card contains four different attributes.
- There are three kinds of each attribute.
  - Colors: red, green, and purple.
  - Shapes: squiggle, diamond, and oval.
  - Fill options: solid, open, and shaded.
  - Number of shapes: one, two, or three
- Below are two examples of a set and by using the multiplication principle we can see how many cards are in a single deck.



$$(3\ colors) * (3\ fill\ options) * (3\ number\ of\ shapes) * (3\ shapes) = 3^4 = 81$$

$81 - (2\text{ cards chosen}) = 79$, $\frac{1}{79}$ *chance of finding a set.*

Question 2: What is the most efficient algorithm to find a set?

```
[ ] #Function 1:
    def all_same_all_not(at_1, at_2, at_3):
      if at_1 == at_2 and at_2 == at_3:
        return True
      elif (at_1 != at_2) and (at_2 != at_3) and (at_3 != at_1):
        return True
      else:
        return False
```

```
[ ] #Function 2:
    def check_table(card1, card2, card3):
      check_color = all_same_all_not(card1.color, card2.color, card3.color)
      check_shape = all_same_all_not(card1.shape, cars2.shape, card3.shape)
      check_number = all_same_all_not(card1.number, card2.number, card3.number)
      check_fill = all_same_all_not(card1.fill, card2.fill, card3.fill)
      return check_color and check_shape and check_fill and check_number
```

Question 3. If we were to add two more Set decks, each containing the same cards as a single deck, how would the game change?

- Let's first ask the question: How many distinct sets can be formed in a regular Set deck?

$$\frac{(81*80)}{3!} = \frac{6480}{6} = 1080 \text{ distinct sets}$$

- If we look at the first two cards, they can either be same or different, but the third card has limited options for what it could be, hence the 81 being multiplied by 80 and divided by three factorial.
- A regular Set deck contains 81 cards, as we know, but by adding two more decks we can conclude the following:

$$81 + 81 + 81 = 243 \text{ cards}$$

$$\frac{(243*242)}{3!} = \frac{58806}{6} = 9801 \text{ distinct sets}$$

- Now that the game has three times the cards, we have changed how it can be played. There are now three of each kind of card, meaning "three of a kind" could now count as a potential set.
- After you choose your first card, you have two choices:
  - Find all three of same exact card.
  - Or find a set (all the same or all different).

$$243 - (2\ exact\ same) = \frac{1}{241}\ chance\ of\ finding\ a\ set.$$

- The chances of finding the exact same card are $\frac{1}{241}$.

$$243 - (2\ different\ or\ same) = \frac{3}{241}\ chance\ of\ finding\ a\ set.$$

- The chances of finding a true set are $\frac{3}{214}$ because three of the same necessary card remain in the deck.
- Since it checks to see if the cards are the same in all ways, the algorithm does not need to be altered in order to find a set.